DESIGN CONCEPTS OF SINGLE DISH TELESCOPES AND PHASED ARRAYS

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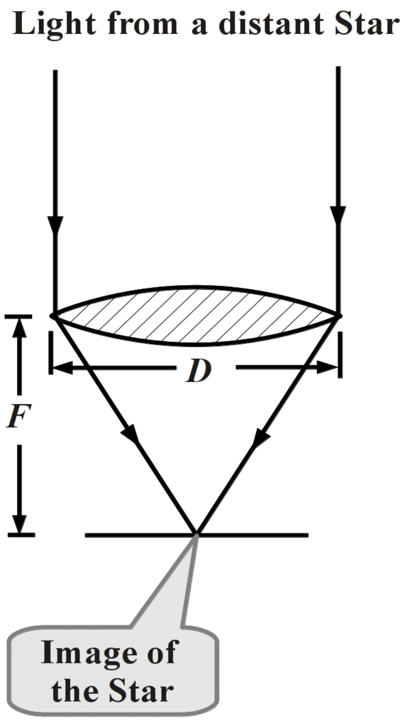
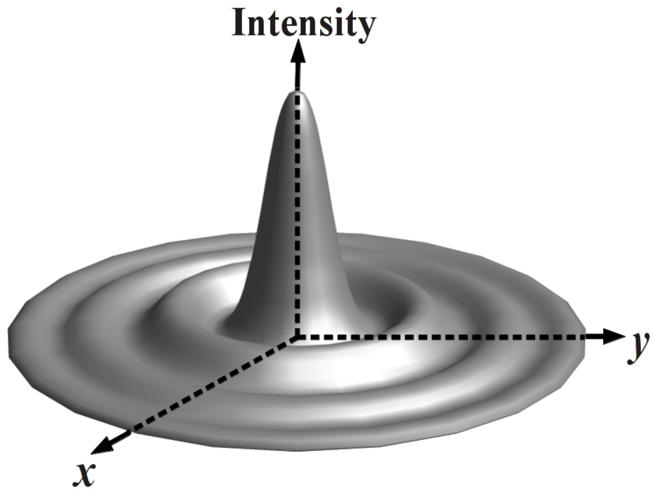
License

Introduction

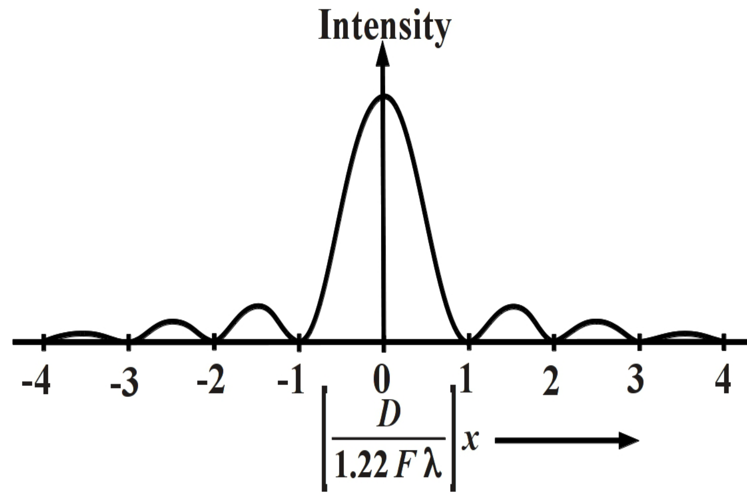
In the last few lectures, we have studied about the basics of the antennas. In this lecture we shall study the design considerations of single dish radio telescopes and construction of arrays. In other words, we shall aim to cover the topics of filled aperture radio telescopes. To begin with we shall start with the angular resolution of a telescope as described by Rayleigh criterion. Then we move on to the design of single dish radio telescopes and aperture systhesis. Later we shall talk about phased antenna arrays.

Raleigh Criterion - I





Point spread function



Point spread function

Rayleigh criterion: Two monochromatic point sources (at same wavelength) can be resolved with a single telescope if the angular separation between the two objects is at least equal to or greater than α' as expressed below:

$$\alpha' = 1.22\frac{\lambda}{D} \approx \frac{\lambda}{D}$$

Here, λ is the wavelength, D is the aperture diameter of the telescope and α' is the minimum angle between the two point sources in radians.

Raleigh Criterion - II

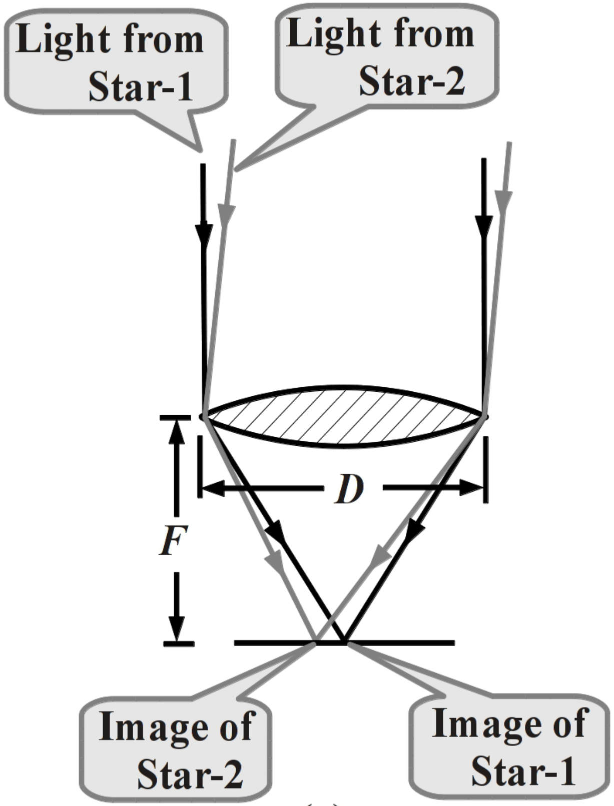
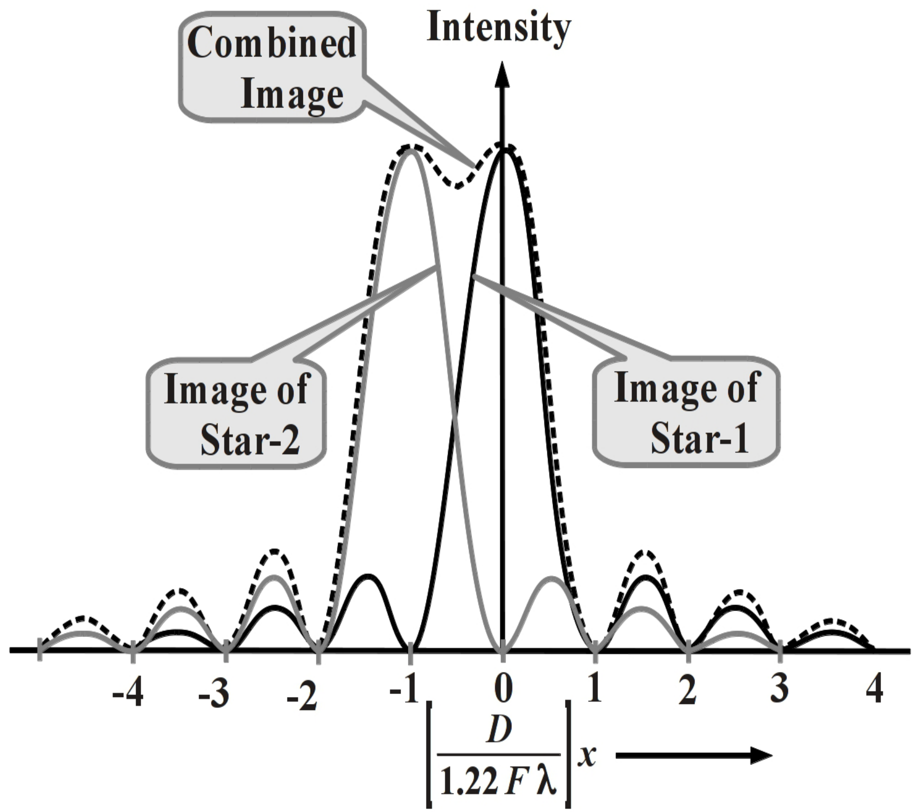


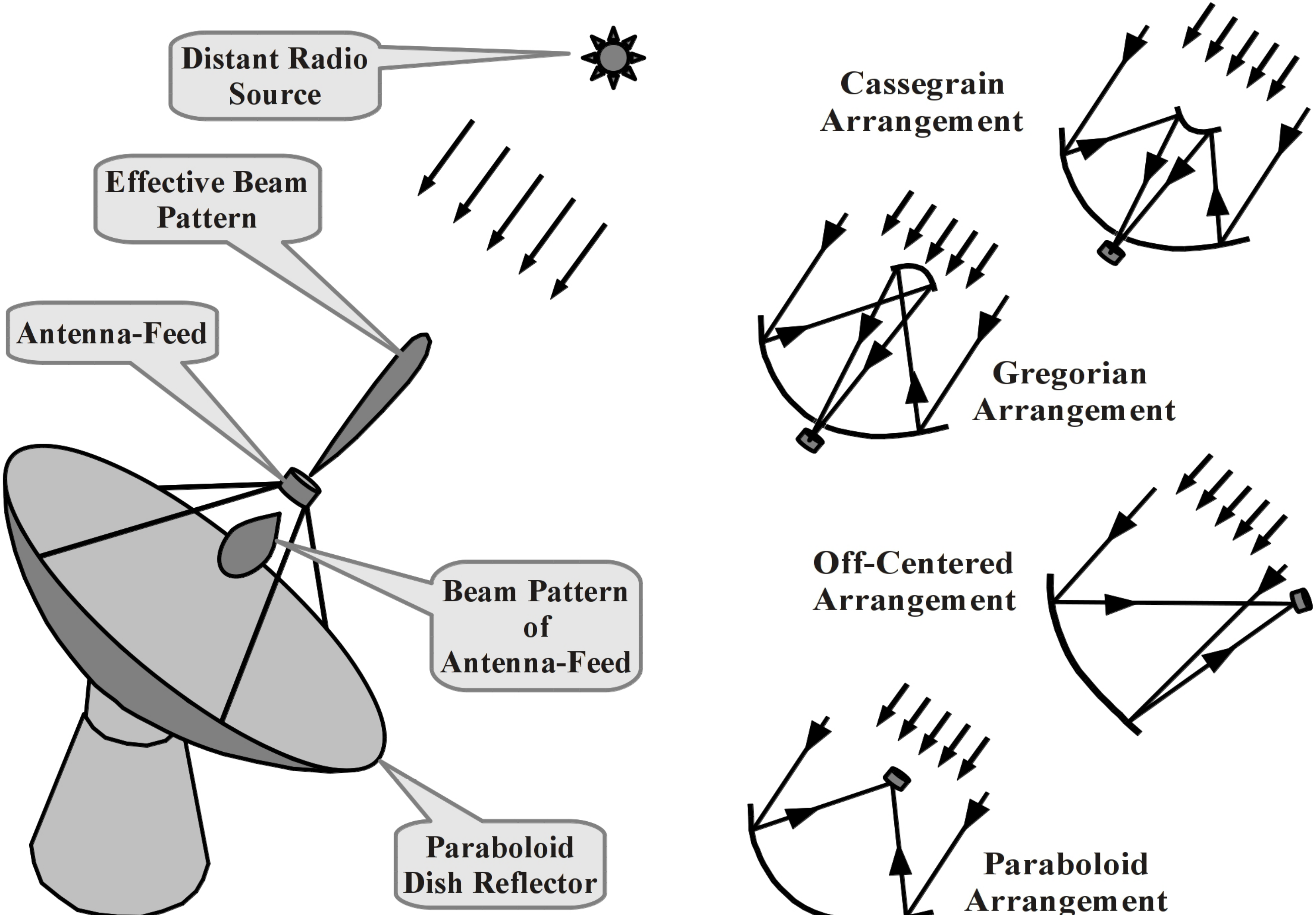
Image from two stars



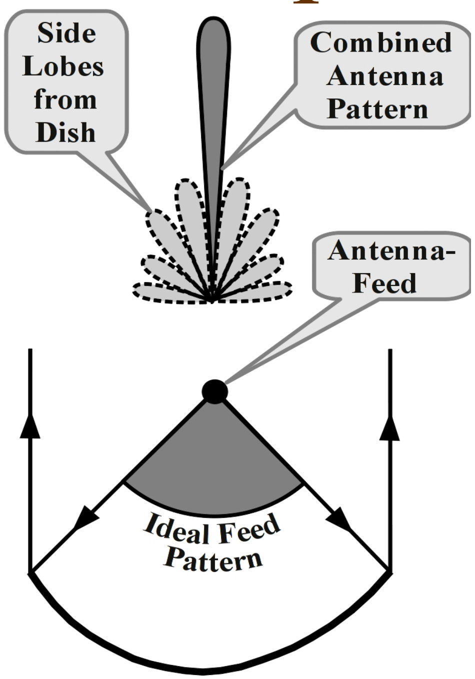
Point spread functions

Note: The diffraction pattern is a function of wavelength. Thus only monochromatic light is assumed for setting Raleigh criterion. If the frequency band-width is increased, spread functions will result for each wavelength and the image will be blurred out.

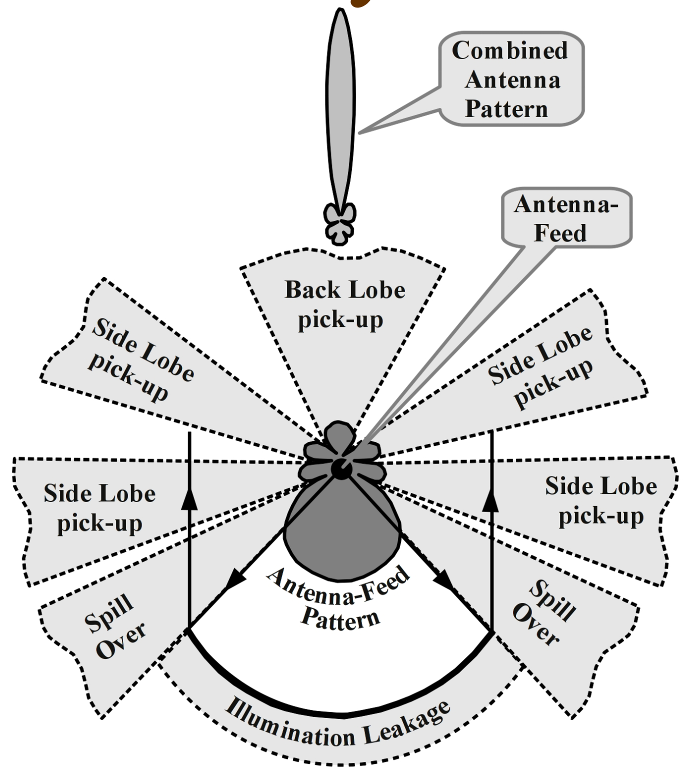
Single Dish Feeding Arrangements



Dish Aperture Illumination by Feed - I



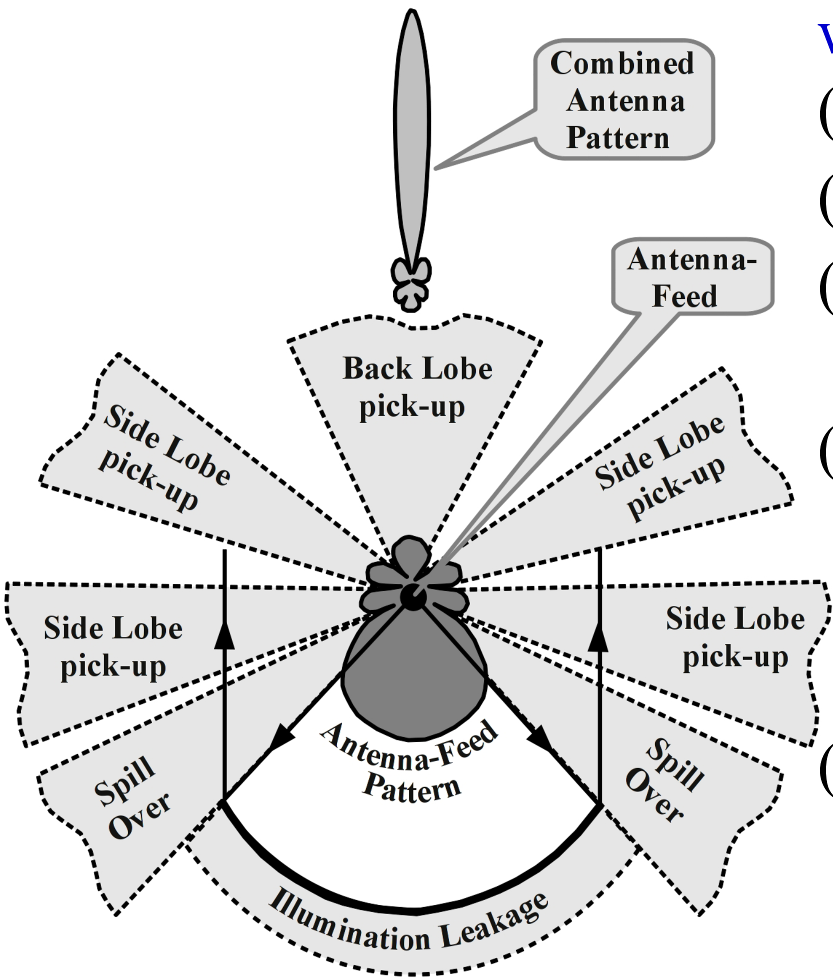
$$\Omega_M \approx \Omega_A = k_B\,\theta_{HP}\,\Phi_{HP}$$



An ideal antenna-feed is capable of illuminating the aperture of the dish uniformly. **Side Effects:** Huge side-lobes are generated due to discontinuity at the edge of the dish.

Actual antenna-feed pattern produces a non uniform illumination of the dish. **Side Effects:** (*i*) spill over, (*ii*) minor lobes pick up (different directions), (*iii*) dish may posses some illumination leakage, (*iv*) combined pattern has side lobes and back lobes.

Dish Aperture Illumination by Feed - II



There are six major factors affecting the performance of the dish antenna system which are:

(*i*) **Antenna efficiency of the feed** (η_a)**.**

(*ii*) **Spill over efficiency** (η_{sp})**.**

(*iii*) **Dish leakage efficiency** (η_{msh})**:** depends on the quality of the mesh.

(*iv*) **Surface smoothness efficiency** (η_{rms})**:** depends on the root mean square variations on the surface of the dish.

(*v*) **Illumination efficiency** (η_{ill})**:** depends on the intensity variation of the signal on the dish.

(*vi*) **Polarization efficiency** (η_{pol})**.**

Dish Aperture Illumination by Feed - III

(*i*) **Antenna efficiency of the antenna-feed**

$$\eta_a = \frac{R_r}{R_r + R_l}$$

R_r – radiation resistance, R_l – loss resistance.

(*ii*) **Spill over efficiency**

$$\eta_{sp} = \frac{Actual\ power\ falling\ on\ the\ Dish - surface}{Total\ power\ radiated\ by\ the\ Antenna - Feed}$$

(*iii*) **Dish leakage efficiency**

$$\eta_{msh} = \frac{Power\ reflected\ by\ Dish\ surface}{Total\ power\ falling\ on\ Dish}$$

(*iv*) **Surface smoothness efficiency**

$$\eta_{rms} = \frac{Actual\ power\ available\ at\ the\ Focus}{Theoretical\ power\ at\ Focus\ (Dish\ surface\ is\ smooth)}$$

(*v*) **Illumination efficiency**

$$\eta_{ill} = \frac{Total\ power\ illuminating\ the\ Dish}{Power\ required\ for\ uniform\ illumination\ at\ peak\ intensity\ of\ Feed}$$

(*vi*) **Polarization efficiency**

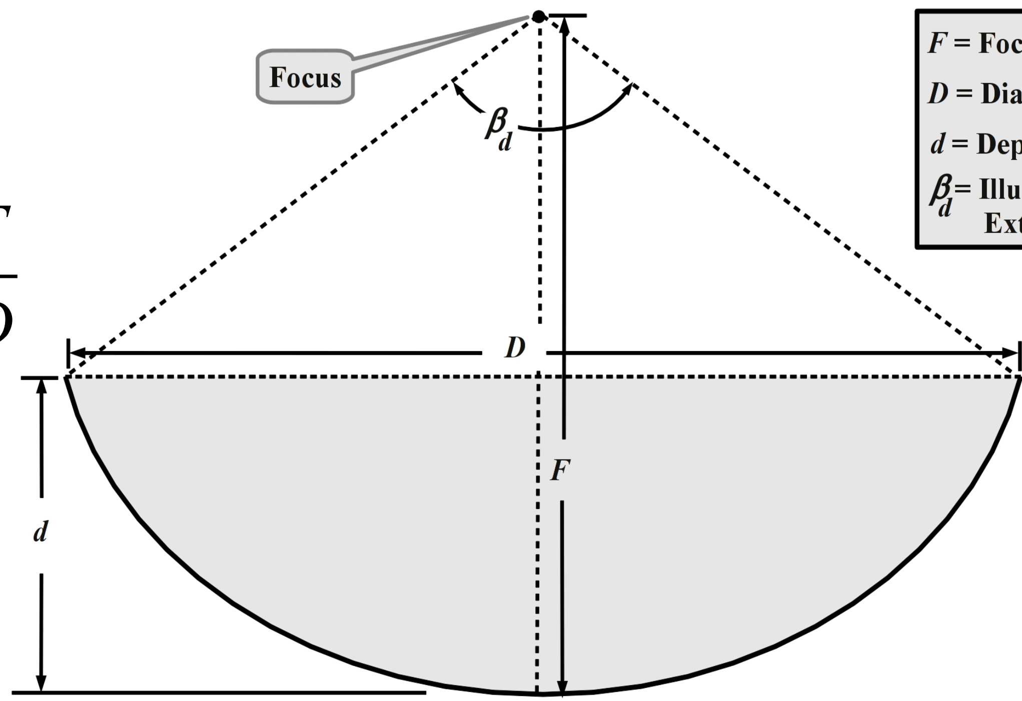
$$\eta_{pol} = \frac{Actual\ power\ received\ by\ a\ matched\ load}{Maximum\ power\ received\ if\ Antenna\ had\ optimum\ polarization\ ability}$$

Analysis of Gain and Aperture of Dish Ant.

Dish diameter $D = \dfrac{F}{K_{FD}}$

where,

$$K_{FD} = \frac{\text{Focal length}}{\text{Diameter}} = \frac{F}{D}$$



Dish depth $d = \dfrac{D^2}{16F}$

Dish illumination extent

$$\beta_d = 2tan^{-1}\left(\frac{D/2}{F-d}\right)$$

Effective gain

$$G_{eff} = \eta_A \left(\frac{\pi D}{\lambda}\right)^2$$

Effective aperture area

$$A_e = \eta_A \left(\frac{\pi D^2}{4}\right) \quad \text{where, } \eta_A < 1$$

Effective HPBW

$$\theta_{effHP} = \sqrt{\frac{40000}{G_{eff}}}$$

Aperture efficiency

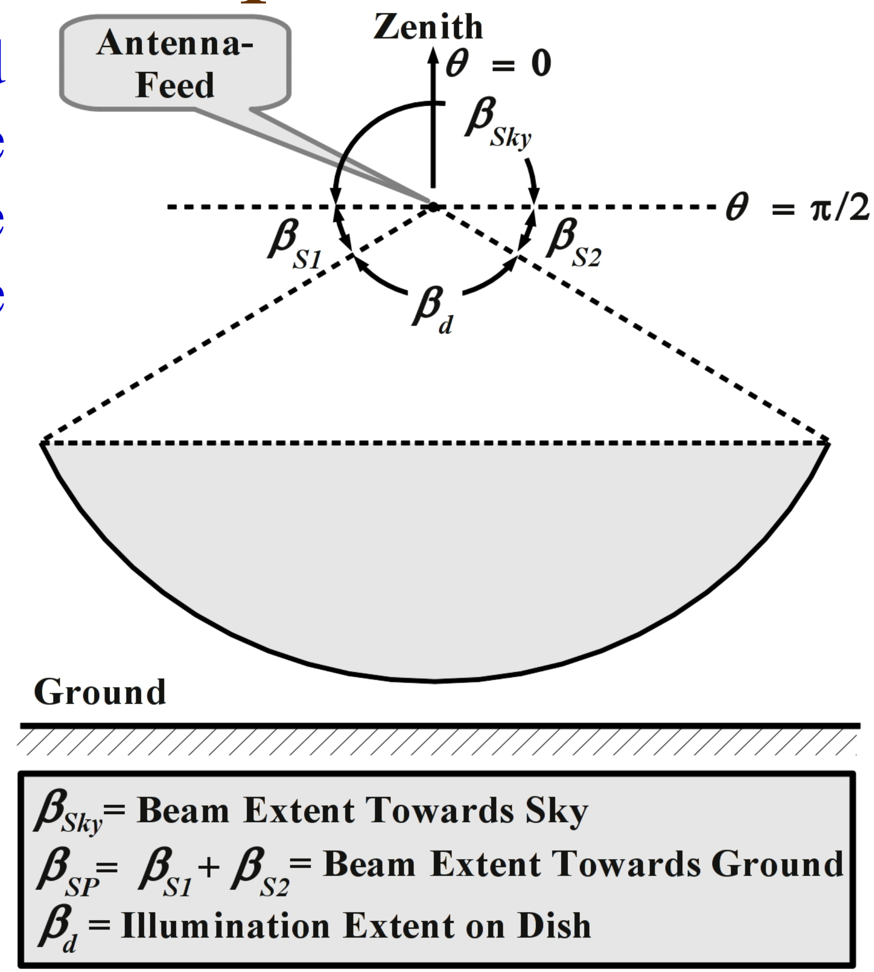
$$\eta_A = \eta_a \, \eta_{sp} \, \eta_{msh} \, \eta_{rms} \, \eta_{ill} \, \eta_{pol}$$

Analysis of Antenna Temperature - I

Consider a paraboloid dish is pointed to an astronomical radio source towards the zenith. Let the source be randomly polarized. We assume the polarization efficiency $\eta_{pol} = 1$.

Contributions to antenna temperature are from (*i*) source, (*ii*) spillover, (*iii*) sky and (*iv*) leakage through the mesh of the dish.

Considering the feed pattern to be cylindrically symmetric around the zenith (*z*-axis), we may define:



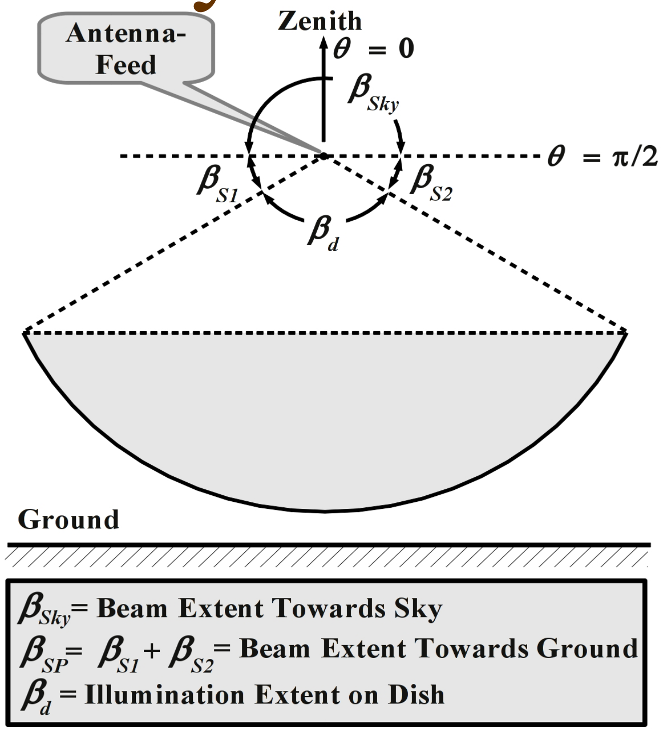
Dish illumination extent β_d

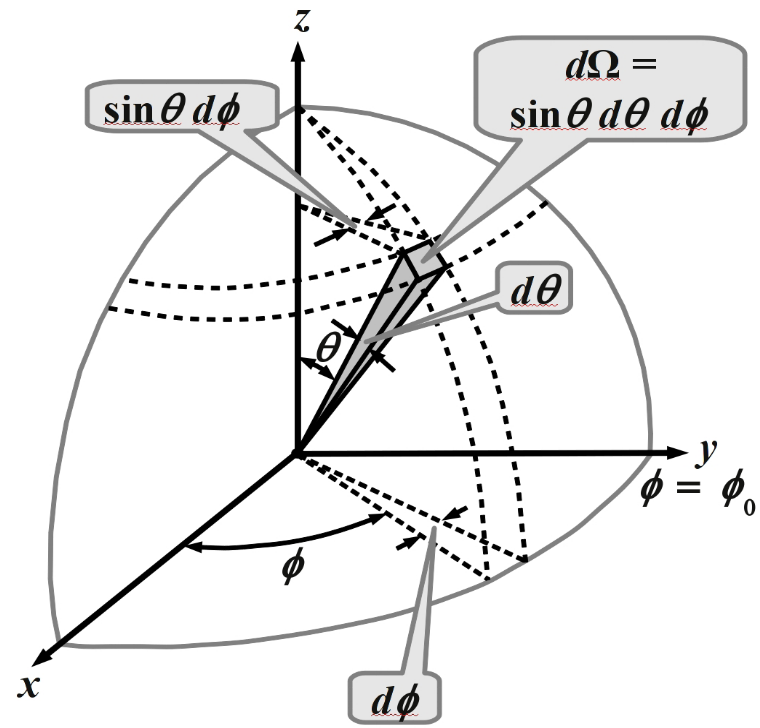
Ground spillover $\beta_{SP} = \beta_{S1} + \beta_{S2}$

Sky angle β_{Sky}

Beam solid angle (dish) $\Omega_{Dish} \approx (\theta_{effHP})^2$

Analysis of Antenna Temperature - II





Let $P_n(\theta,\varphi_0)$ be the normalized feed-power-pattern on a plane at $\varphi = \varphi_0$.

Effective beam solid angle Ω_{12} subtended between θ_1 and θ_2 is given as:

$$\Omega_{12} = \int_0^{2\pi} \int_{\theta_1}^{\theta_2} P_n(\theta, \phi) \sin\theta \, d\theta \, d\phi$$

Simplifying, we get $\Omega_{12} = 2\pi \int_{\theta_1}^{\theta_2} P_n(\theta, \phi) \sin\theta \, d\theta$

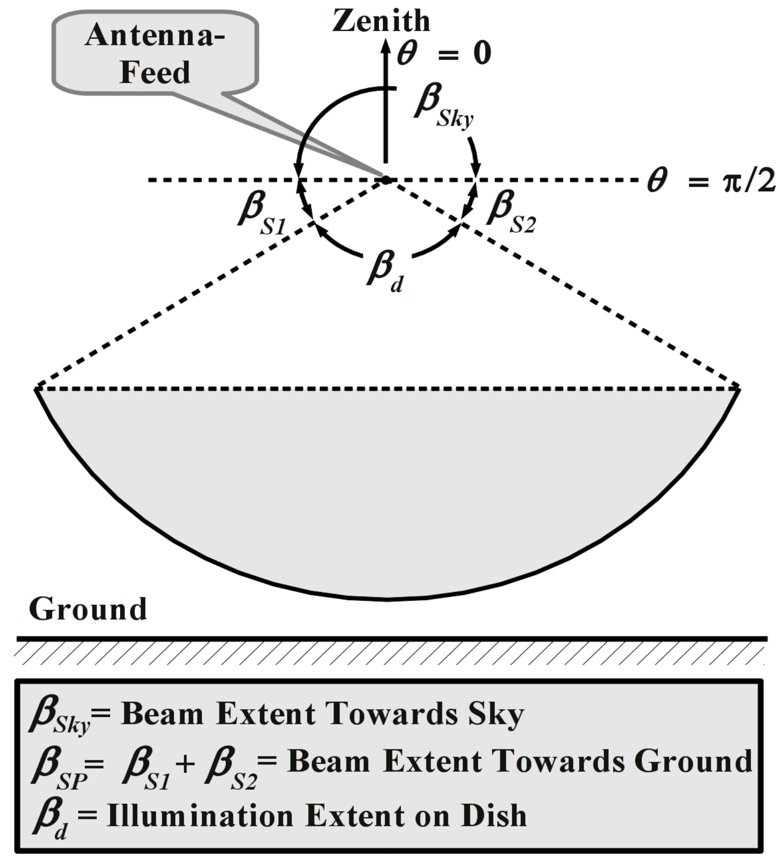
We shall use this generic integral to calculate various solid angles next.

Analysis of Antenna Temperature - III

Effective beam solid angle of the feed facing the sky (including side lobes and back lobe) is given as:

$$\Omega_{Sky} \approx 2\pi \left[\int_0^{\frac{\pi}{2}} P_n(\theta, \phi_0) \sin\theta \, d\theta \right]$$

where, φ_0 is the plane of paper.



Effective beam solid angle of feed illuminating the dish

$$\Omega_d \approx 2\pi \left[\int_{\frac{\pi}{2}+\frac{\beta_{SP}}{2}}^{\pi} P_n(\theta, \phi_0) \sin\theta \, d\theta \right]$$

Note that, $\beta_{S1} = \beta_{S2} = \dfrac{\beta_{SP}}{2}$

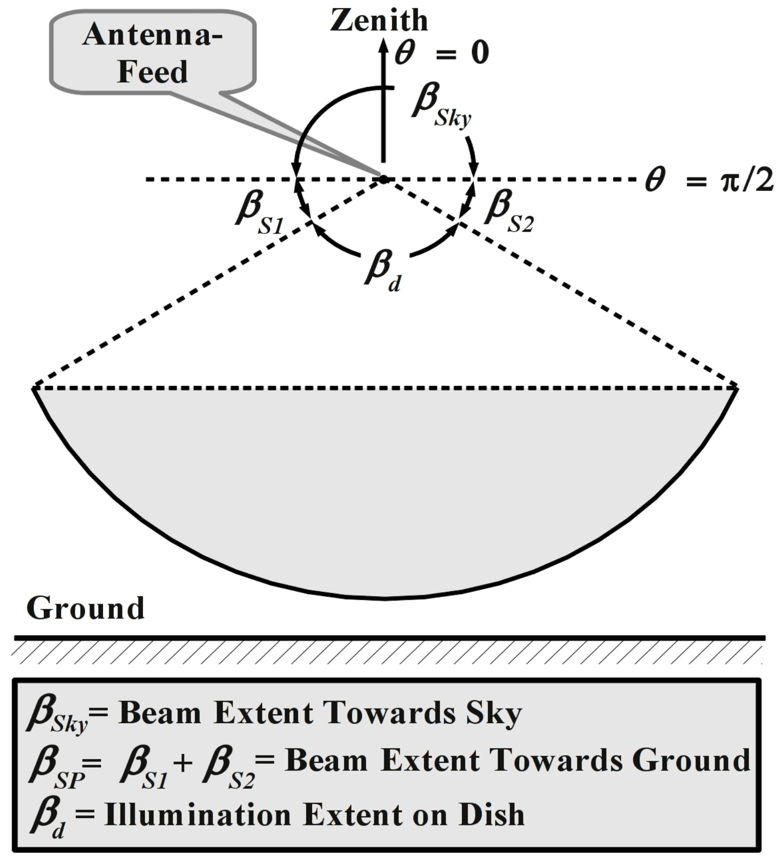
Effective beam solid angle of feed causing spill over

$$\Omega_{SP} \approx 2\pi \left[\int_{\frac{\pi}{2}}^{\frac{\pi}{2}+\frac{\beta_{SP}}{2}} P_n(\theta, \phi_0) \sin\theta \, d\theta \right]$$

Analysis of Antenna Temperature - IV

Effective beam solid angle (spill over)

$$\Omega_{SP} \approx 2\pi \left[\int_{\frac{\pi}{2}}^{\frac{\pi}{2}+\frac{\beta_{SP}}{2}} P_n(\theta, \phi_0) \sin\theta \, d\theta \right]$$



Let the source be randomly polarized. We also assume that the polarization efficiency of the antenna is $\eta_{pol} = 1$.

Temperature contribution to the antenna system from the dish alone is given as:

$$T_{Dish} = \eta_{msh}\eta_{rms}\Omega_s \overline{T}_{Src} + (\Omega_{Dish} - \Omega_s)\, T_{Sky}$$

where,

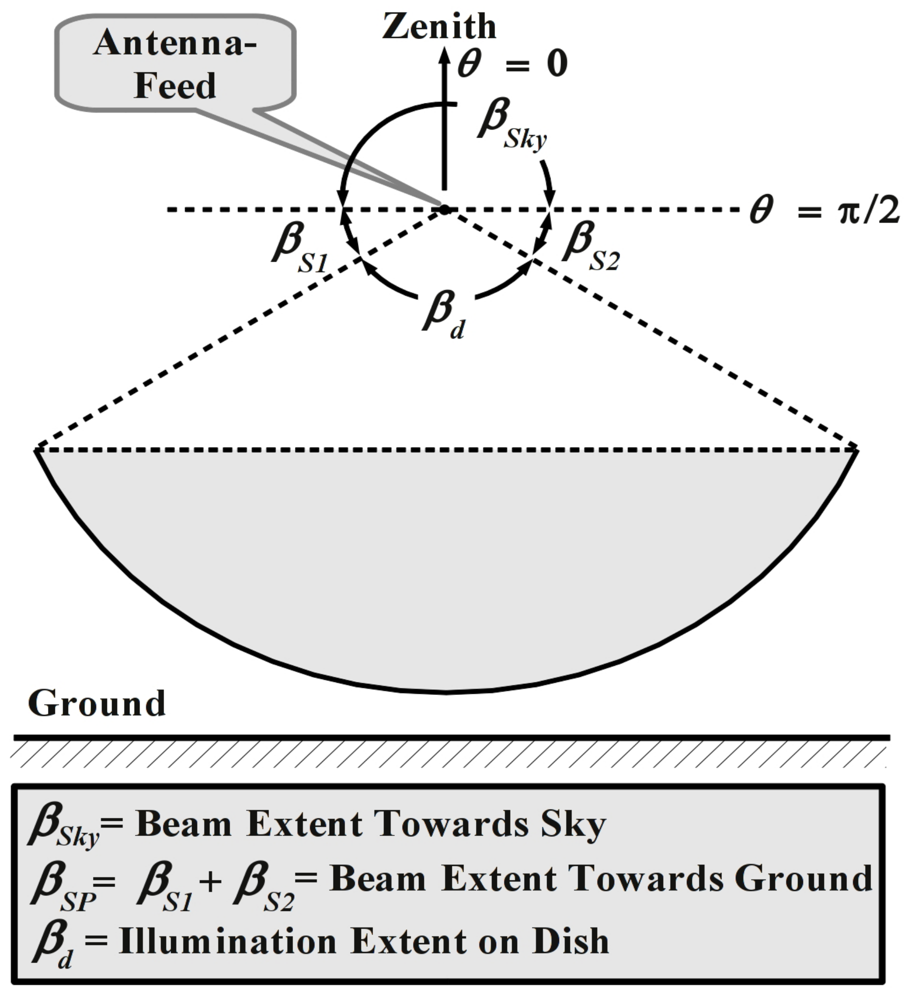
beam solid angle subtended by the source is Ω_s

and, the mean temperature of the source over Ω_s is $\overline{T}_{Src}$.

Analysis of Antenna Temperature - V

The overall antenna temperature T_A is a result of the contributions from

(*i*) source,
(*ii*) spillover,
(*iii*) sky, and
(*iv*) leakage through the dish.



The temperature contributions can be subdivided into three groups

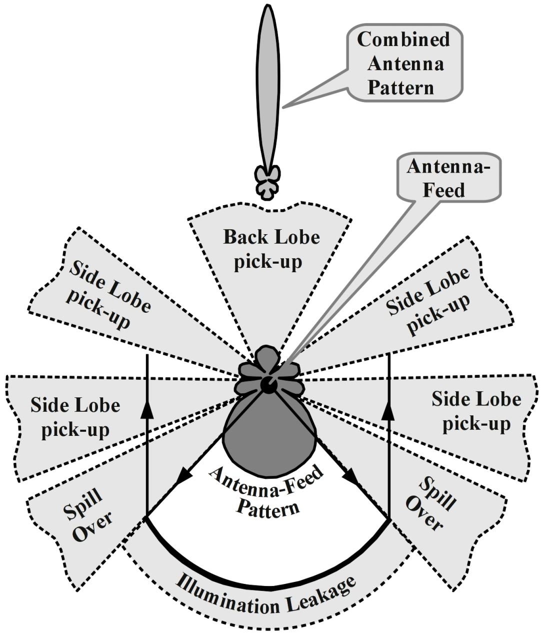
(*i*) from sky $T_{SkyPickup}$
(*ii*) from ground spill over $T_{GndPickup}$
(*iii*) from ground due to mesh leakage $T_{GndLeakage}$

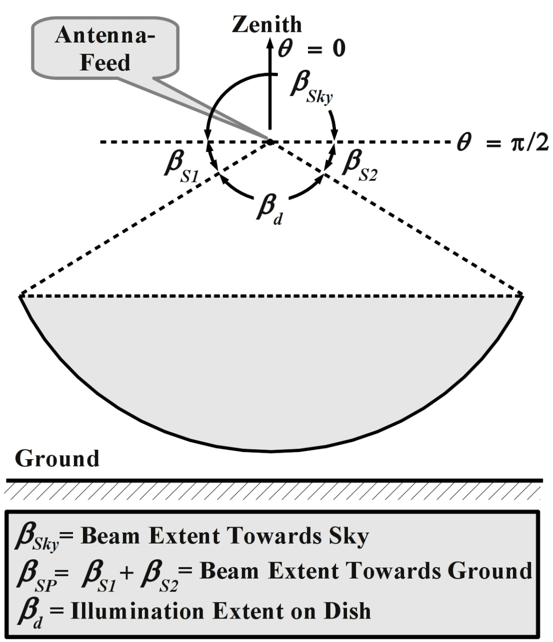
Antenna temp. $T_A = T_{Dish} + \eta_a \left[T_{SkyPickup} + T_{GndPickup} + T_{GndLeakage}\right]$

where, $T_{SkyPickup} = \Omega_{Sky} T_{Sky}$ $\quad T_{GndPickup} = \Omega_{SP} T_{Gnd}$

$$T_{GndLeakage} = \Omega_d T_{Gnd} (1 - \eta_{msh})$$

Reduced Aperture Illumination





Although intensity of feed-pattern is less across the spill-over, it is reasonably higher than the side lobes. It can pickup large noise from ground. Solution is to reduce the feed spill over by narrowing its the main lobe. It is known as ***reduced aperture illumination***.

Drawbacks: it reduces η_{ill} and thus reduces aperture efficiency η_A.

The dish coordinates keep changing with time. As it moves away from the zenith the ground contribution reduces in one side and increases on the other side since more side lobes point to the ground on one side than the other. But the dish illumination by the feed remains constant. The mesh efficiency η_{msh} must be kept close to unity for this purpose.

Designing a Single Dish Radio Telescope

Design specifications required

(i) Angular size of the object

(ii) Brightness of the source

(iii) Brightness of the sky

(iv) Frequency or wavelength

(v) Bandwidth

Design steps

1. Find the minimum diameter D' (m) of a cent percent efficient aperture telescope. [α_s = source extent in radian] $$D' = 1.22\frac{\lambda}{\alpha_s}$$

2. Calculate D (m) with aperture efficiency η_A between 0.4 and 0.5 as: $$D = D'\sqrt{\eta_A}$$

3. Calculate the effective aperture area A_e (m^2) $$A_e = \frac{\pi D^2}{4}\eta_A$$

4. A fairly good receiver has a temperature T_R between 35 and 55 K. Calculate system temperature T_{Sys} $$T_{Sys} = T_A + T_R$$

5. Calculate the incremental temperature ΔT (K) $$\Delta T = \left(T_{Sys}\right)_{src} - \left(T_{Sys}\right)_{sky}$$

The design is likely to succeed only if ΔT emerges as a positive quantity!!

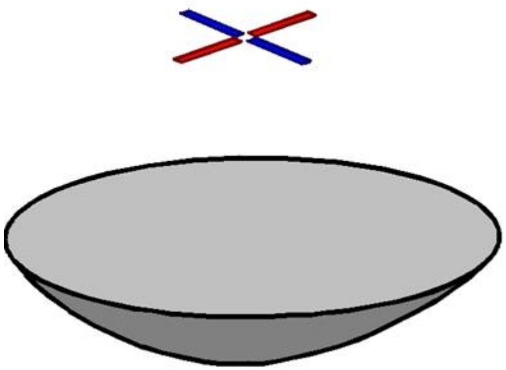
Antenna Feeds

Several type of antenna-feeds are available today. Each has its own advantages and disadvantages in respect of ***gain***, ***bandwidth***, ***efficiency*** and ***side lobes*** etc.

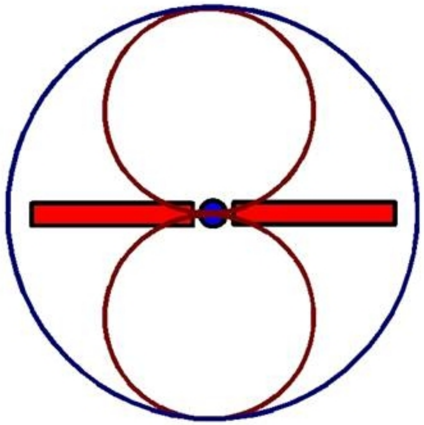
We begin with a simple dipole feed for creating a background of understanding the concepts. Then we move out to ultra wide band antenna-feeds, since they look more promising for the future ultra wide band technology of the radio telescopes. Finally we discuss the horn antenna feeds. These are listed below:

(*i*) Cross-Dipole Antenna Feed
(*ii*) Log Periodic Dipole Arrays (LPDA)
(*iii*) Non-Planar Ultra Wide Band Log Periodic Antenna Feeds
(*iv*) Planar Log-Periodic Ultra Wide Band Antenna-Feeds
(*v*) Horn antenna feeds

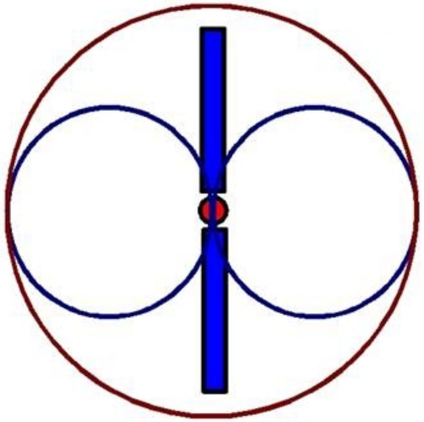
Cross dipole Feed



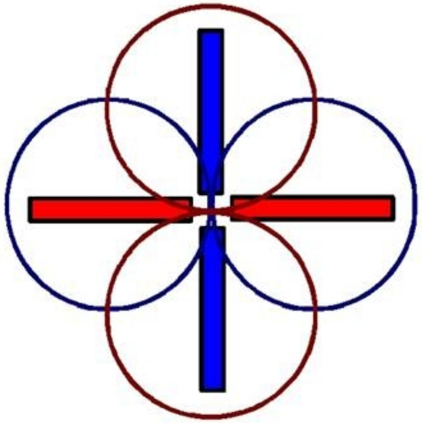
A dipole-cross is mounted at the focus of a parabolid dish.



Radiation pattern of the red dipole as observed from the top.



Radiation pattern of the blue dipole as observed from the top.



Radiation patterns of the dipole-cross as observed from top.

Cross dipole Feed

Disadvantages of the cross dipole-feed

The dipole-feed is inefficient in several respects as listed below:

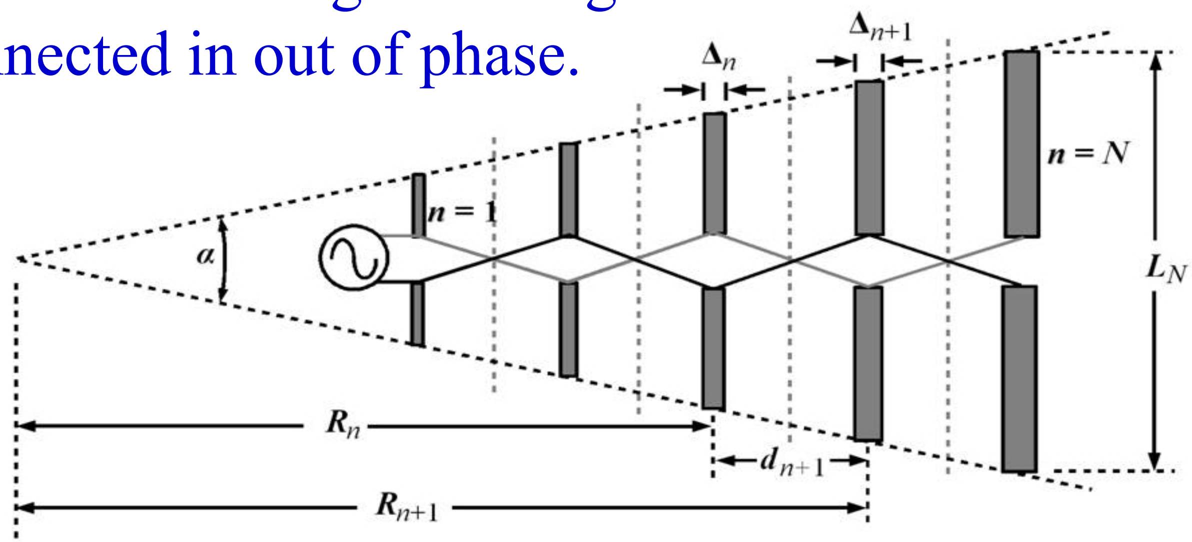
(*i*) narrow band width,
(*ii*) large back lobes and
(*iii*) poor directivity.

The back lobe may be reduced by using a reflector at the back of cross. But most of the advantages of using a dish are spoiled if a dipole-feed is used.

Log Periodic Dipole Array design - I

These consist of dipoles of different lengths arranged in ascending order and connected in out of phase.

The length L_n, spacing d_n and the thickness Δ_n of the dipoles increases with distance starting from the smallest dipole.



N = No. of dipole elements.

Ratio of adjacent elements $\tau = \dfrac{R_n}{R_{n+1}} = \dfrac{L_n}{L_{n+1}} = \dfrac{\Delta_n}{\Delta_{n+1}} < 1, \quad \text{where, } 1 \le n \le N$

Spacing factor $\sigma = \dfrac{d_n}{2L_n}$ where, $d_n = R_{n+1} - R_n$

Apex angle $\alpha = \tan^{-1}\left(\dfrac{1-\tau}{4\sigma}\right)$ Also, $\tan\left(\dfrac{\alpha}{2}\right) = \dfrac{0.5L_n}{R_n} = \dfrac{0.5L_{n+1}}{R_{n+1}}$

Log Periodic Dipole Array design - II

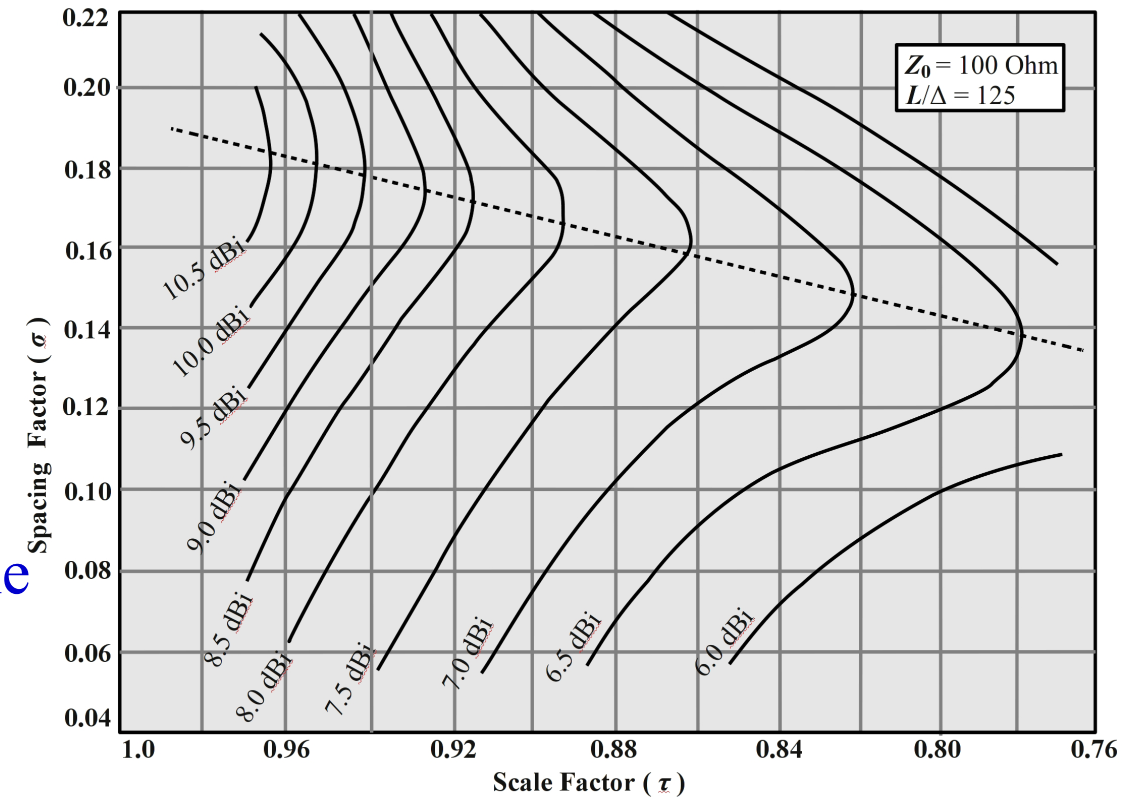
Select a design curve based on required gain. Dotted line gives optimized values of σ.

Calculate length of smallest dipole L_1 as:

$$L_1 = 0.5 \frac{c}{\nu_{\max}}$$

Using L_1 with σ calculate the lengths of remaining dipoles.

Calculate τ from two adjacent element lengths.

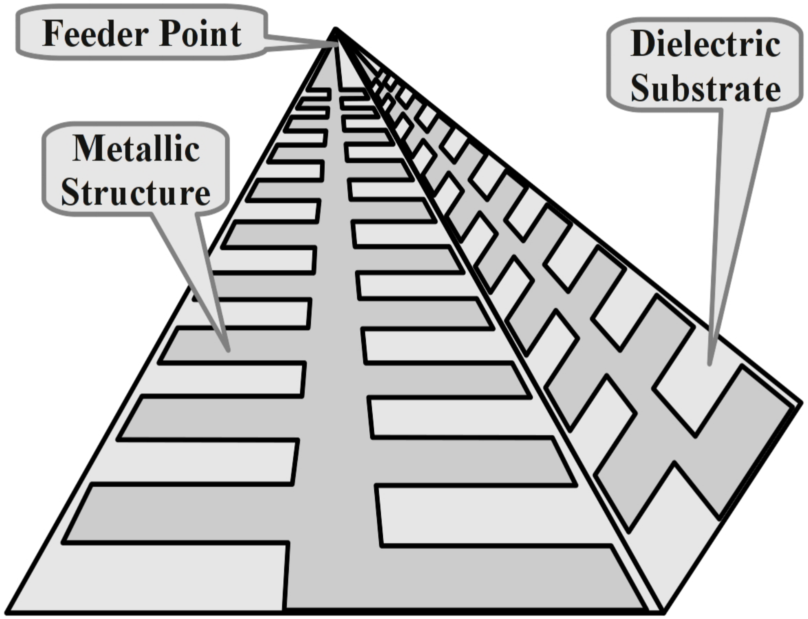


Gains with Z_{of} = 100 ohm, L/Δ = 125.

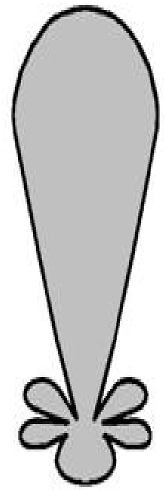
Characteristic impedance of LPDA $Z_0 = Z_{0f}\left(1 + \frac{Z_{0f}}{2Z_{0\Delta}X}\right)$ where, $X = \frac{8\tau\sigma}{1+\tau}$

$Z_{0\Delta}$ = characteristic impedance of monopole of diameter Δ when posited between ground planes positioned on dotted lines (previous page).

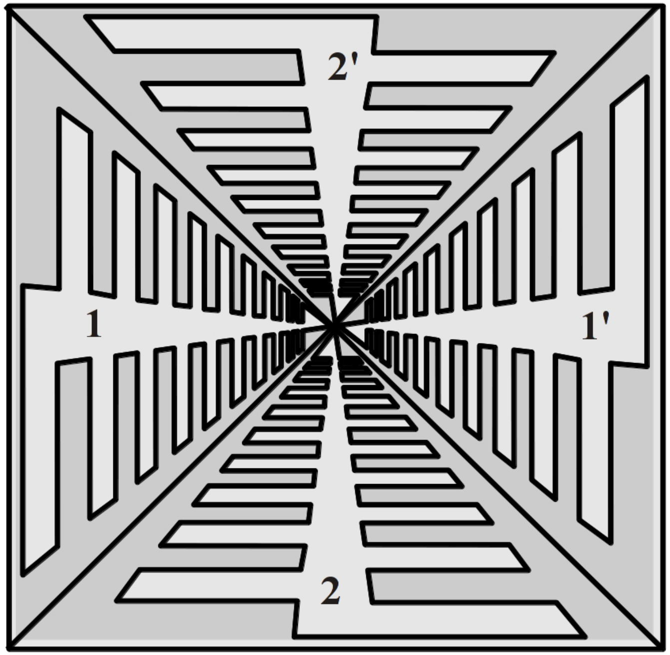
Non-Planar UWB LP Antenna Feeds - I



Isometric view of the non planar log periodic antenna-feed.

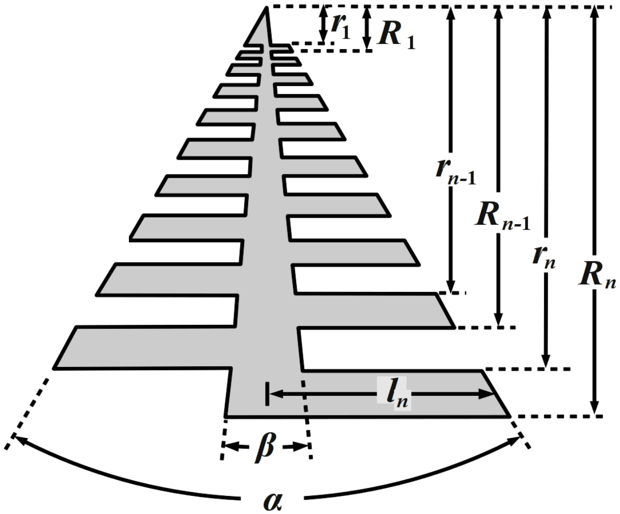


Radiation pattern orientation.

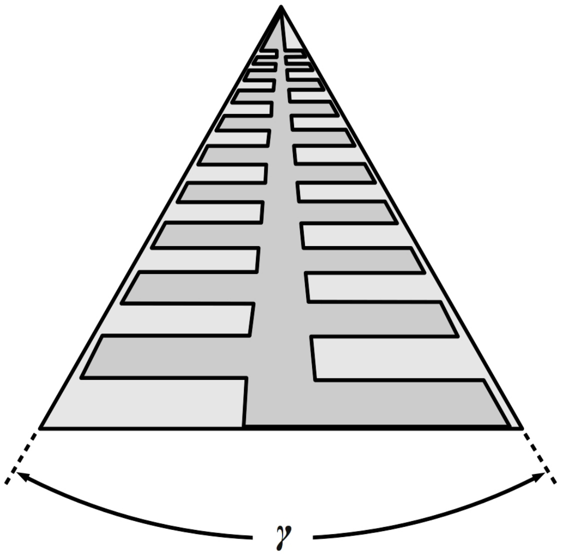


Top view of the non planar log periodic antenna-feed.

Non-Planar UWB LP Antenna Feeds - II



Design dimensions.



Angle between two halves of an antenna.

Frequency of n^{th} dipole $\nu_n = \dfrac{c}{4l_n}$

where, n is the no. of dipoles, and c is speed of light.

Ratio of edge distances τ between two adjacent dipoles

$$\tau = \frac{R_{n-1}}{R_n} = \frac{r_{n-1}}{r_n} = \frac{l_{n-1}}{l_n} < 1$$

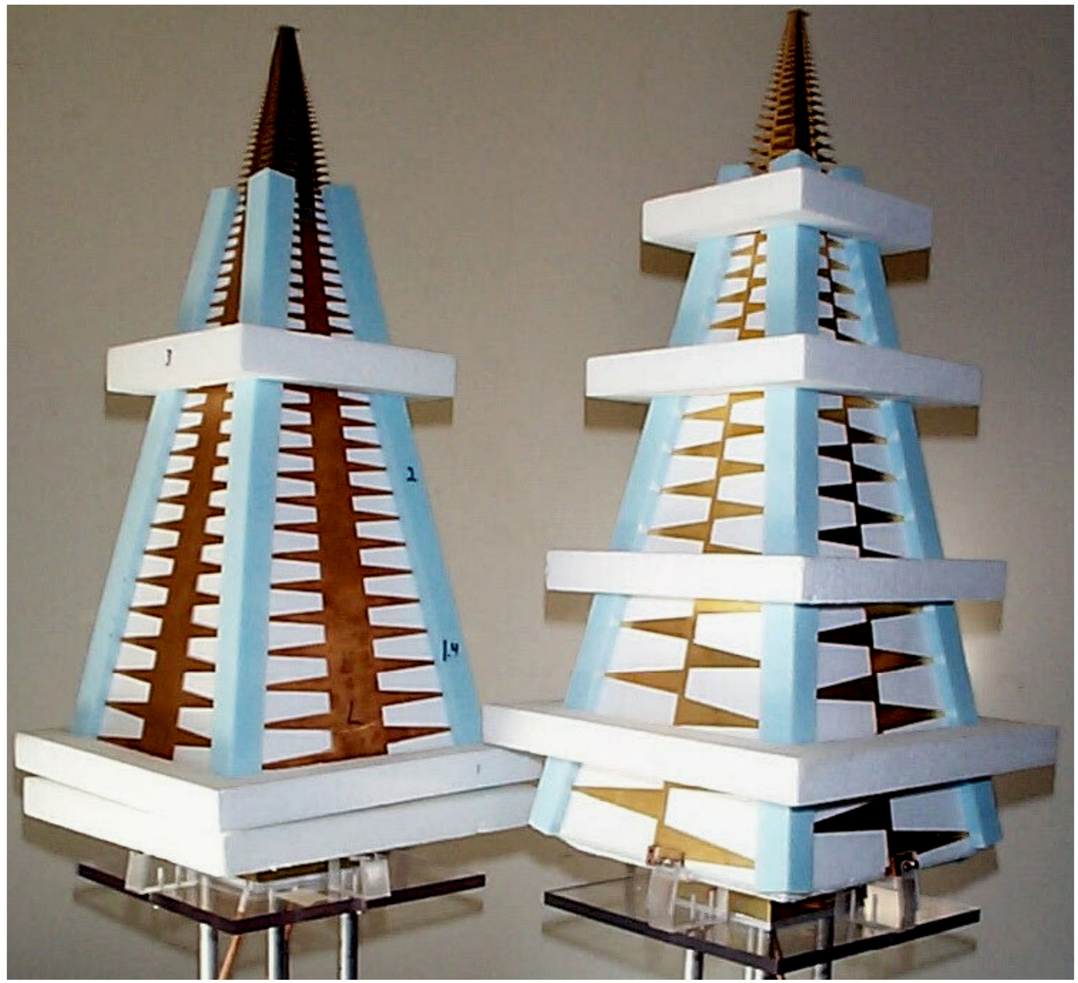
Also, $\tau = \dfrac{\nu_2}{\nu_1} = \dfrac{\nu_3}{\nu_2} = \ldots\ldots < 1$

where, $\nu_1 > \nu_2 > \nu_3 \ldots.$

Again, $\tau^{n-1} = \dfrac{\nu_2}{\nu_1} \times \dfrac{\nu_3}{\nu_2} \times \dfrac{\nu_4}{\nu_3} \ldots. \times \dfrac{\nu_n}{\nu_{n-1}} = \dfrac{\nu_n}{\nu_1}$

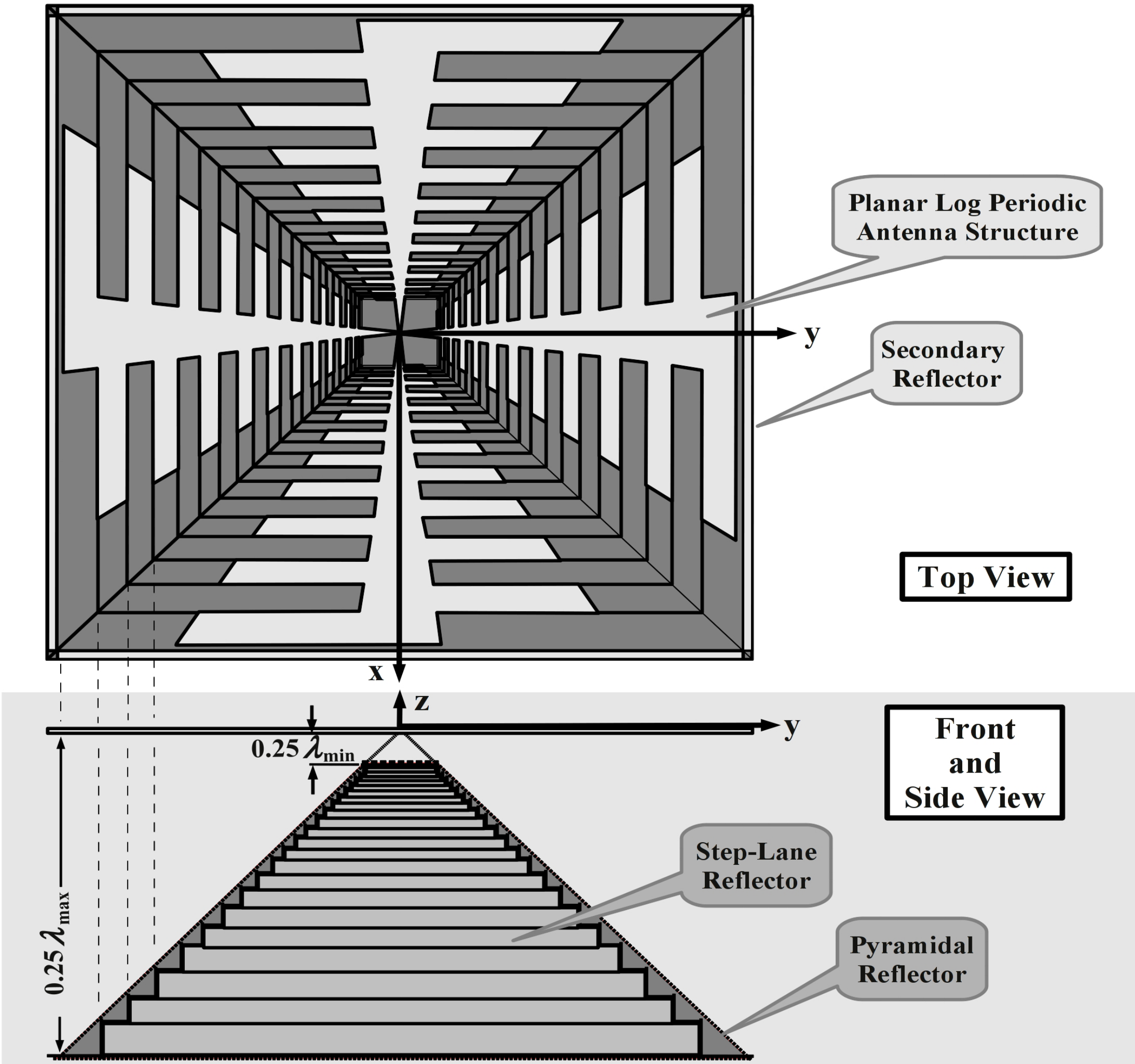
Antenna impedance changes from 70 ohms for $\gamma = 30^o$ to nearly 180 ohms for $\gamma = 180^o$.

Non-Planar UWB LP Antenna Feeds - III



Prototypes of non planar log periodic antenna-feeds for integration with cryogenic amplifier. Courtesy: University of California, Radio Astronomy Lab, Berkeley.

Planar UWB LP Antenna Feeds - I



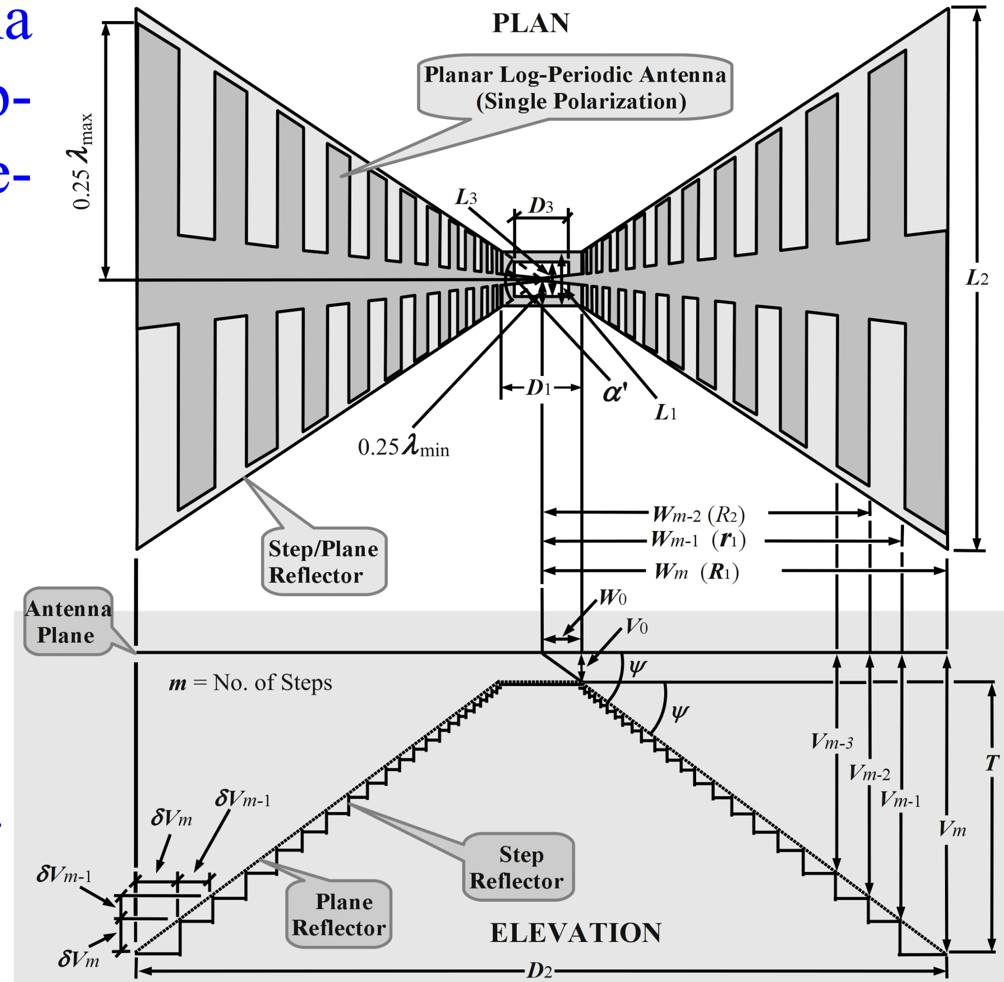
(*i*) Step-lane reflector feed (thick lines)

(*ii*) Pyramidal reflector feed (dotted lines)

The secondary reflector is optional: boosts gain at lower frequencies. Antenna can be fabricated on (*i*) PCB or (*ii*) a metallic sheet shaped and fixed on a low ε_r thin dielectric surface for mechanical support. To further reduce the effect of the dielectric, slots can be made in the dielectric between the dipoles. This also reduces the air resistance.

Planar UWB LP Antenna Feeds - II

Consider a single polarized antenna associated with either a step-reflector (dark lines) or a plane-reflector (dotted lines).



λ_{min} - from smallest dipole.

λ_{max} - from largest dipole.

$G_{LPA+Refl}$ - required gain.

$$G_{LPA} = p\,G_{LPA+Refl} \quad \text{where}, p \approx 0.5$$

Reflector dimensions come from the planar antenna. Each half dipole elements posses two long edges. Let k represent the edge-numbers. The smallest edge resides at $k = 0$, and the largest edge reside at $k = m$, where m is the total number of steps (including top and bottom) if a step-reflector is used.

Planar UWB LP Antenna Feeds - III

$V_0, V_1, \ldots, V_{m-1}, V_m$ - vertical distances between dipole edges and the reflector.

ψ - angle between antenna and reflector.

From this geometry we get:

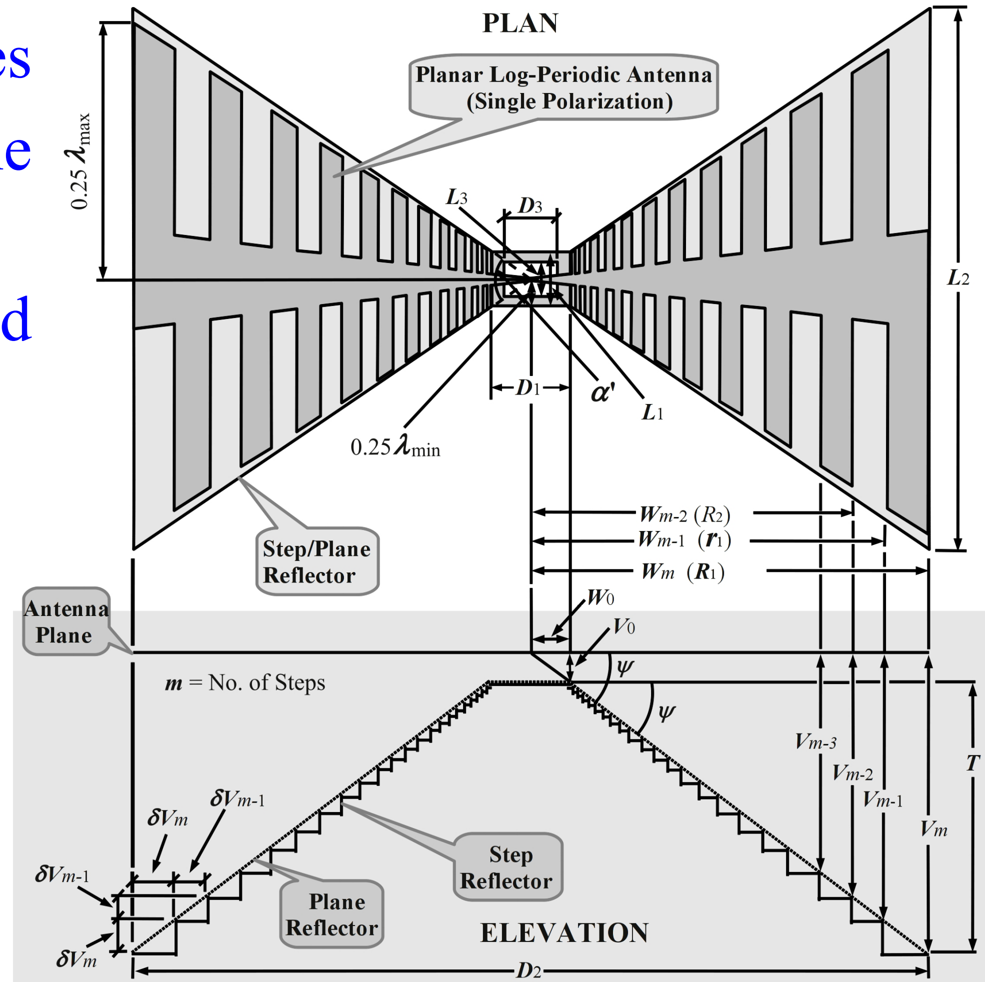
$$tan\Psi = \left(\frac{\delta V_k}{\delta W_k}\right) \quad k = 0, 1, .., m-1, m$$

$$tan\Psi = \left(\frac{V_k}{W_k}\right) \quad k = 0, 1, ..., m-1, m$$

$$V_0 = 0.25\lambda_{min} \qquad V_m = 0.25\lambda_{max}$$

$$D_1 = \left(\frac{\lambda_{min}}{2\tan(0.5\alpha)}\right) \qquad D_2 \geq 2W_m$$

$$D_2 \geq \left(\frac{\lambda_{max}}{2\tan(0.5\alpha)}\right) \quad 0 \leq D_3 \leq 2W_0$$



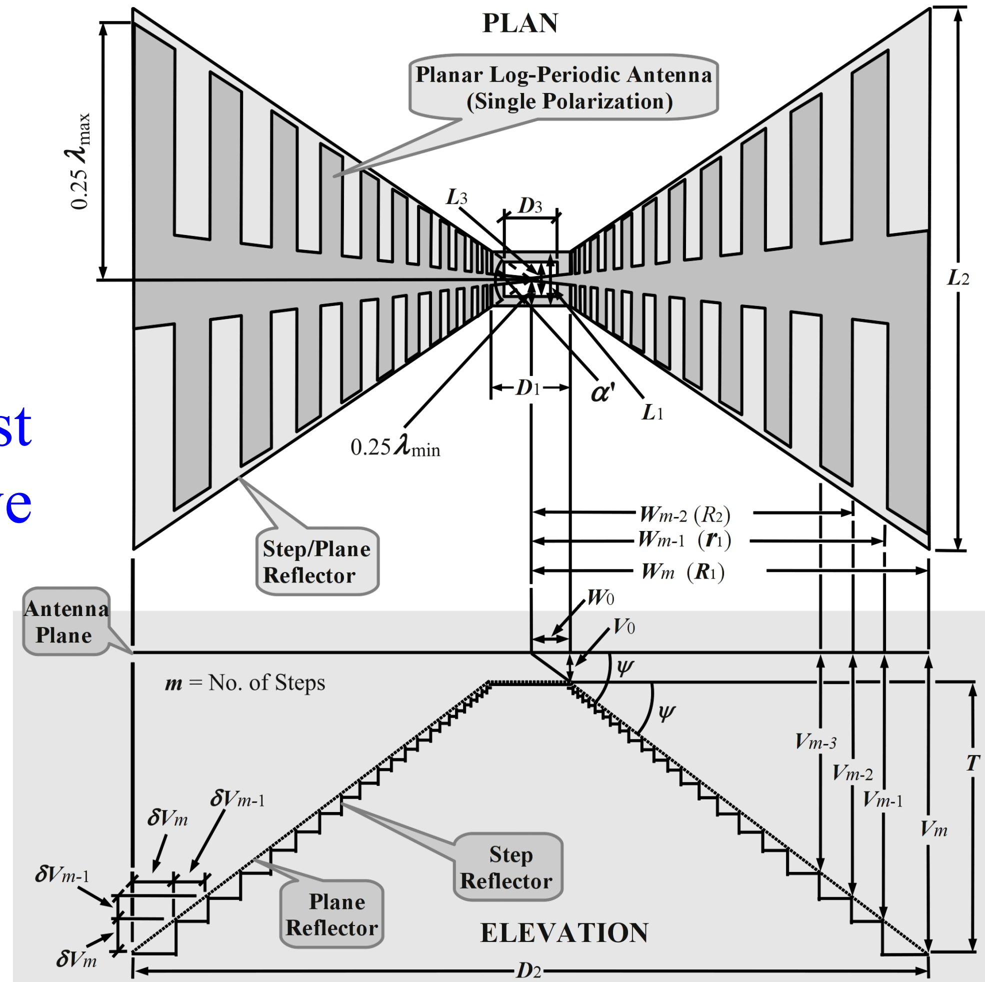
Planar UWB LP Antenna Feeds - IV

$$T = 0.25\left(\lambda_{\max} - \lambda_{\min}\right)$$
$$L_1 \geq 0.5\lambda_{\min}$$
$$L_2 \geq 0.5\lambda_{\max}$$
$$0 \leq L_3 \leq 2V_0$$

Projected reflector angle $\alpha' \geq \alpha$

To reduce the back-lobes at highest frequencies, L_3 and D_3 should have minimized dimensions.

Relations of geometric ratio τ of the planar antenna with reflector:

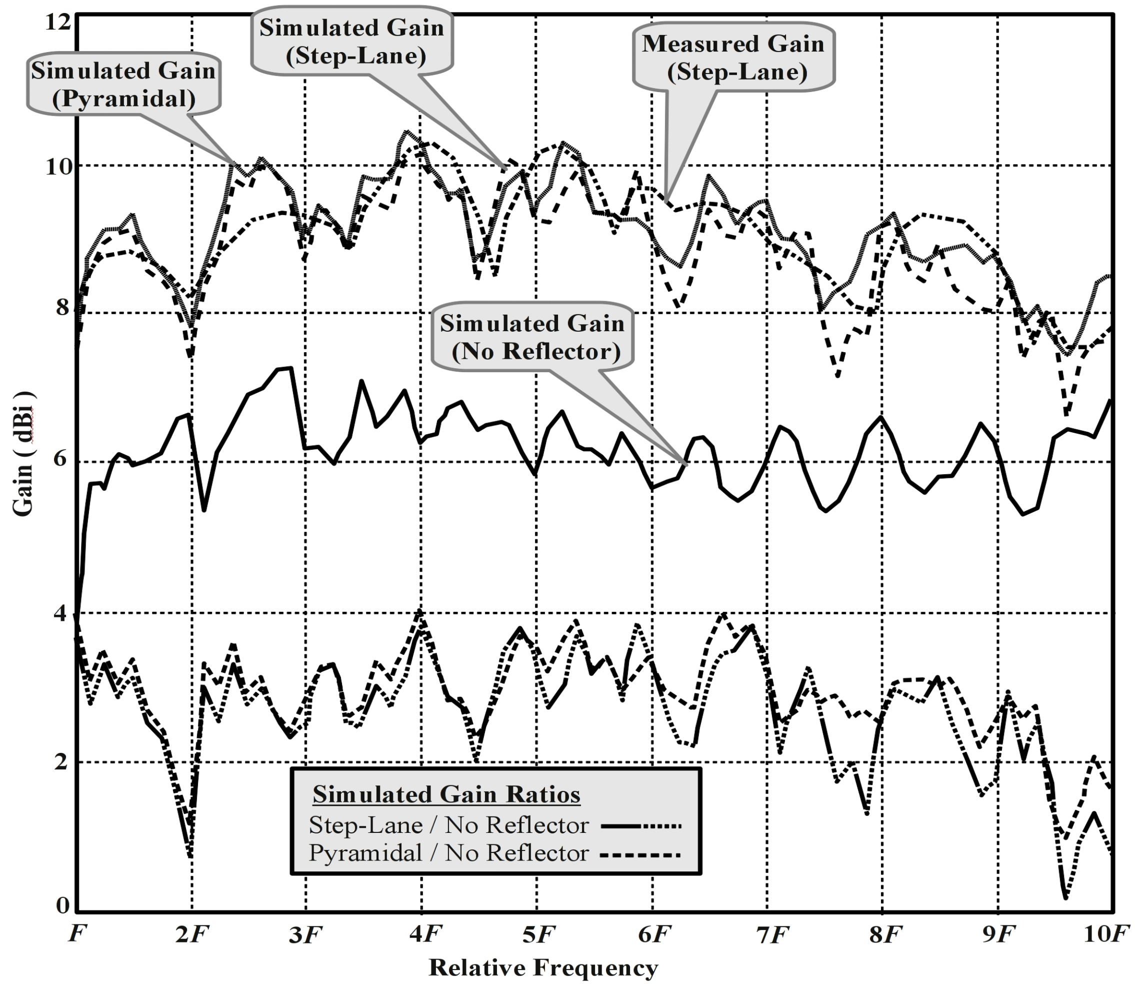


$$\tau = \left(\frac{W_{k-1}}{W_k}\right)^2 \quad k = 0, 1, \ldots, m-1, m$$

$$\tau = \left(\frac{V_{k-1}}{V_k}\right)^2 \quad k = 0, 1, \ldots, m-1, m$$

$$\tau^m = \left(\frac{V_0}{V_m}\right)^2 \qquad \tau^m = \left(\frac{W_0}{W_m}\right)^2$$

Planar UWB LP Antenna Feeds - V



$$G_{LPA} \leq G_{LPA+Refl} \leq 2.65\, G_{LPA}$$

Simulated and measured antenna gains and gain ratios as functions of frequency for various configurations. The electromagnetic contributions from the secondary reflector are included.

Horn Antenna feeds - I

Horn antennas are operated at microwave frequencies. They can form narrow beams and can operate as single aperture antenna. They are also used as antennafeeds for dish antenna systems. The horn antenna is a flared wave guide. Basically, there are two types of horns:

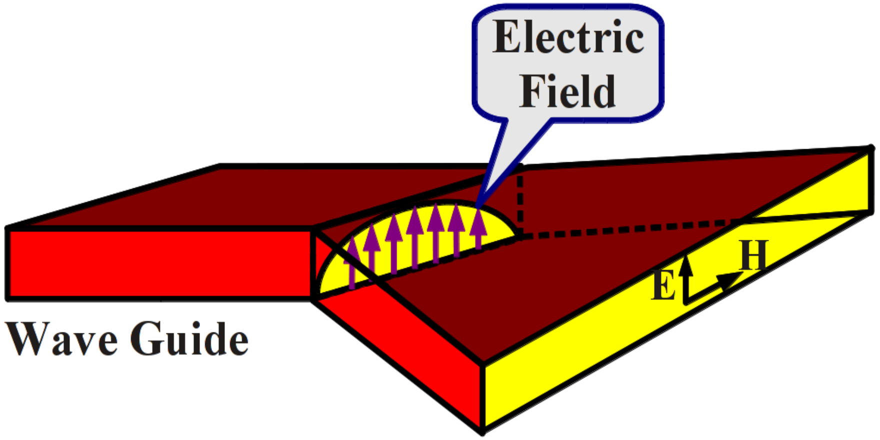
(*i*) Rectangular aperture horn – has rectangular aperture.

(*ii*) Conical aperture horn – has circular aperture.

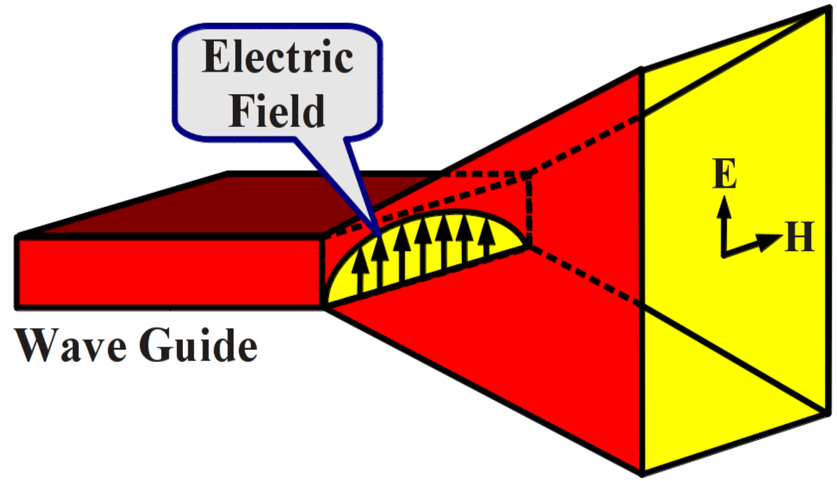
The flaring of a rectangular horn can be in E-plane, or in H-plane, or in both. These horns are respectively named as ***E-plane sectoral***, ***H-plane sectoral*** and ***pyramidal***. Excitations are in TE_{10} mode. Pyramidal horns are popular, since beam narrowing is possible in both directions. Thus pencil beams can be formed.

Here we show the design equations of the pyramidal horn which are based on the design equations of H-plane and E-plane sectoral horns.

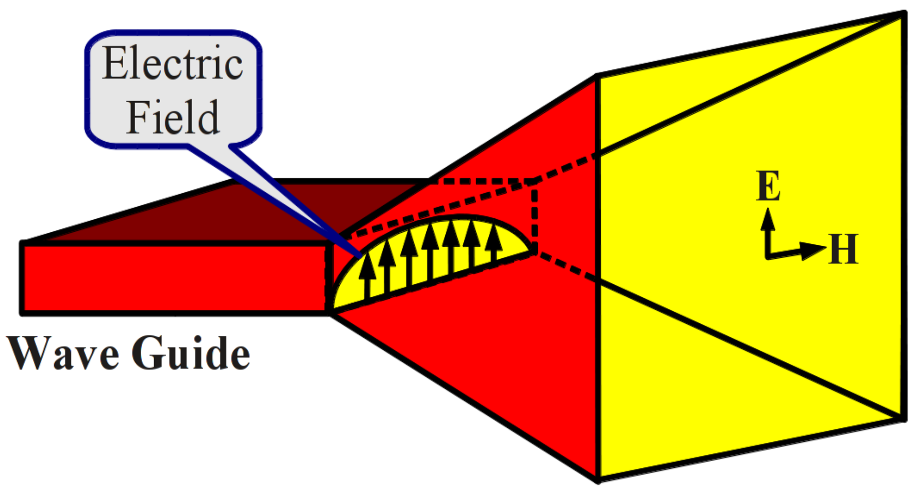
Horn Antenna feeds - II



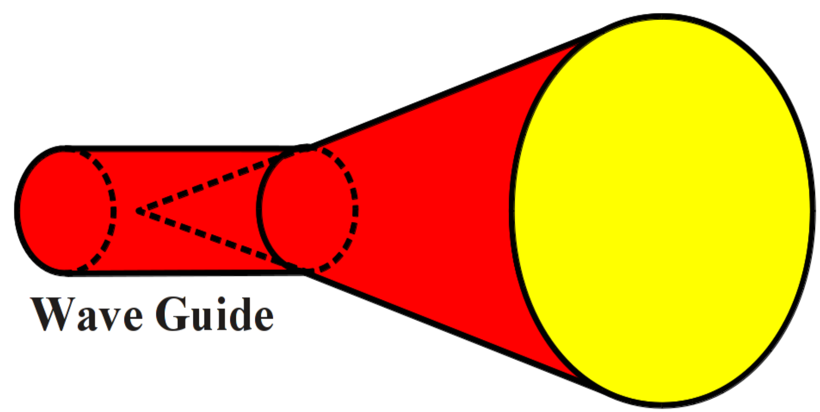
(a) H-Plane Sectoral Horn



(b) E-Plane Sectoral Horn



(c) Pyramidal Horn



(d) Conical Horn

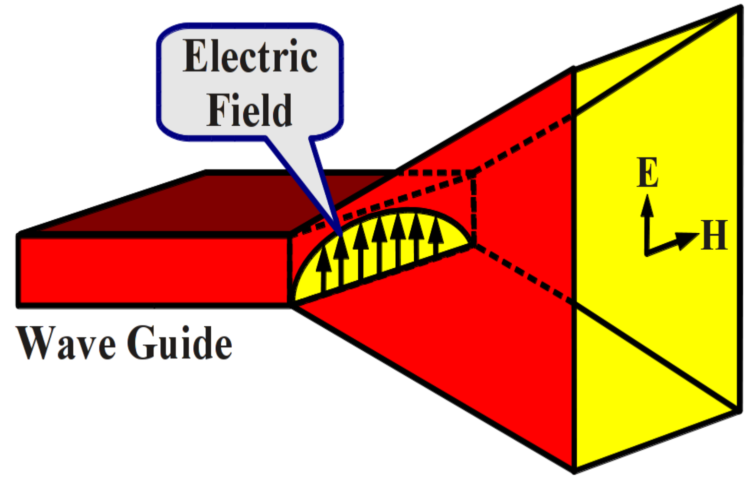
Horn Antenna: Pyramidal Horn Design - I

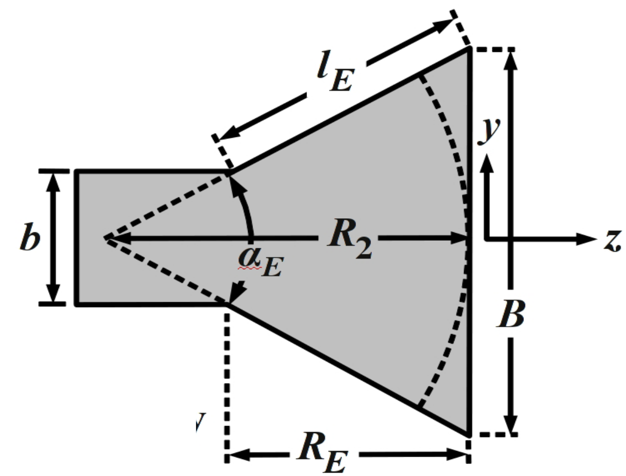
Consider the **E**-plane sectoral horn. From the E-Plane geometry we find:

$$l_E^2 = R_2^2 + \left(\frac{B}{2}\right)^2$$

$$\alpha_E = \tan^{-1}\left(\frac{B}{2R_2}\right)$$

$$R_E = (B-b)\sqrt{\left(\frac{l_E}{B}\right)^2 - \frac{1}{4}}$$





E-Plane geometry

The electric field lies parallel to *y*-axis. It's intensity varies along x-axis. It is expressed as:

$$E_{ay} = E_0 \cos\left(\frac{\pi x}{a}\right) \exp\left[-j\left(\frac{\beta}{2R_2}\right)y^2\right]$$

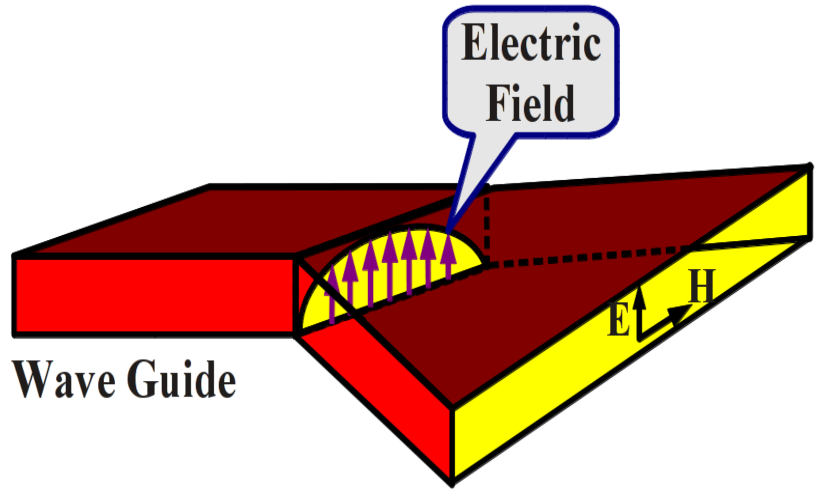
Horn Antenna: Pyramidal Horn Design - II

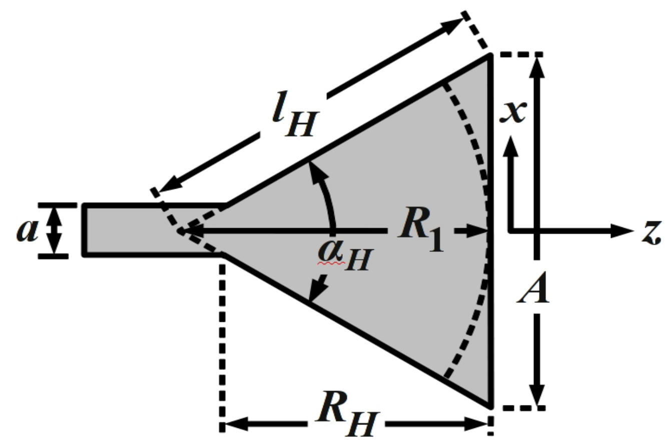
Now consider the **H**-plane sectoral horn. From the H-Plane geometry we find:

$$l_H^2 = R_1^2 + \left(\frac{A}{2}\right)^2$$

$$\alpha_H = \tan^{-1}\left(\frac{A}{2R_1}\right)$$

$$R_H = (A-a)\sqrt{\left(\frac{l_H}{A}\right)^2 - \frac{1}{4}}$$





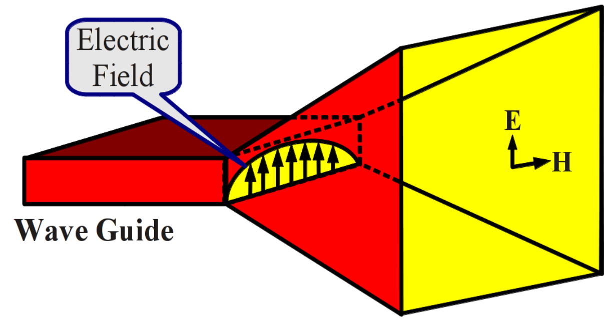
H-Plane Geometry

In this case too, the electric field lies parallel to *y*-axis and it's intensity varies along *x*-axis. It is expressed as:

$$E_{ay} = E_0 \cos\left(\frac{\pi x}{A}\right)\ \exp\left[-j\left(\frac{\beta}{2R_1}\right)x^2\right]$$

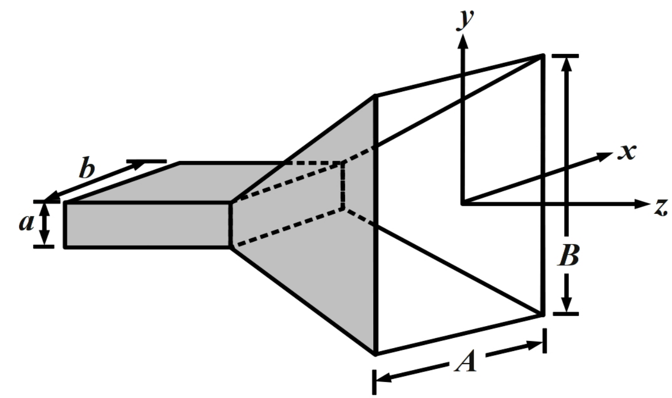
Horn Antenna: Pyramidal Horn Design - III

The pyramidal horn is composed of E-plane and H-plane sectoral horns. We obtained their respective electric field equations as:



$$E_{ay} = E_0 \cos\left(\frac{\pi x}{a}\right) \ \exp\left[-j\left(\frac{\beta}{2R_2}\right) y^2\right]$$

$$E_{ay} = E_0 \cos\left(\frac{\pi x}{A}\right) \ \exp\left[-j\left(\frac{\beta}{2R_1}\right) x^2\right]$$



Isometric Geometry

From above two equations, we may write the electric field E_{ay} of a pyramidal horn as:

$$E_{ay} = E_0 \cos\left(\frac{\pi x}{A}\right) \ \exp\left[-j\left(\frac{\beta}{2}\right)\left(\frac{x^2}{R_1} + \frac{y^2}{R_2}\right)\right]$$

Directivity G_D of pyramidal horn is: $G_D = \left(\frac{4\pi}{\lambda^2}\right) A_e = \left(\frac{4\pi}{\lambda^2}\right)(A\,B\,\eta_A)$

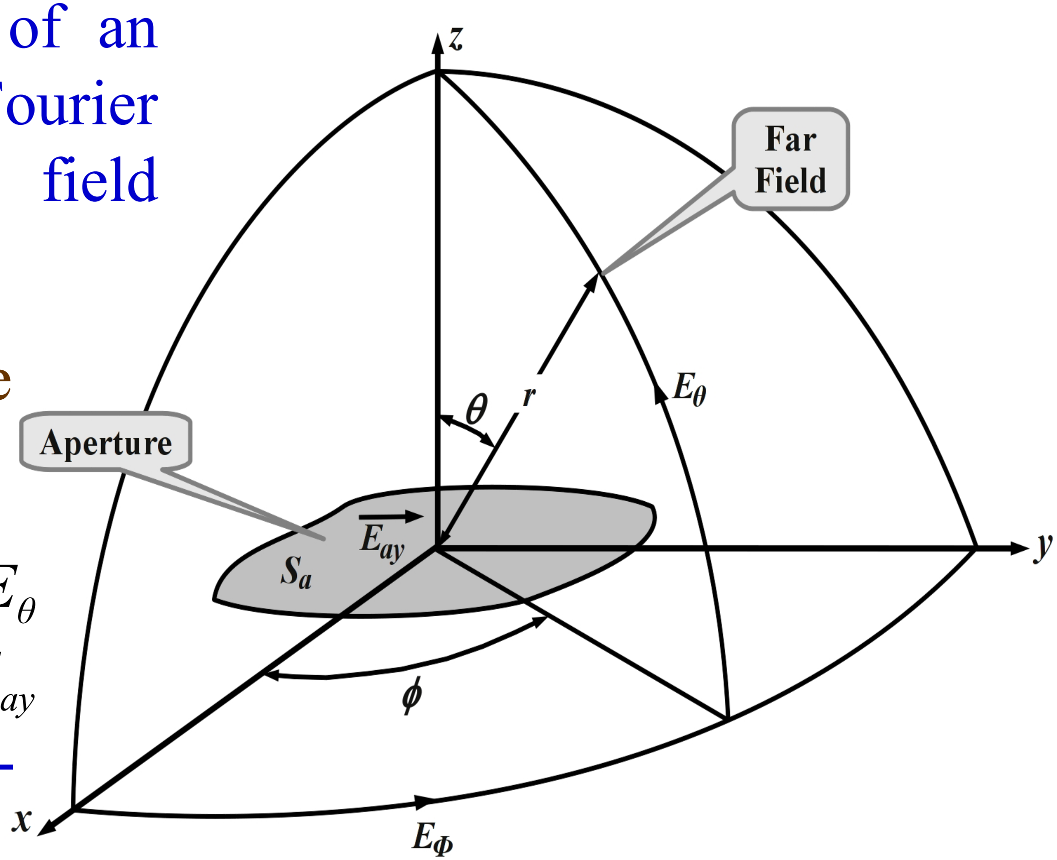
where, A_e is its effective aperture area and η_A is aperture efficiency. For good designs, η_A lies between 0.5 and 0.7.

Aperture and Far Field Relationship

The spatial far field pattern of an antenna is proportional to the Fourier transform of the frequency field pattern at the antenna aperture.

Let electric field E_{ay} on aperture is only along the *y*-axis.

Far electric field components E_θ and E_φ are related with E_{ay} through 2-D Fourier tranforms:-

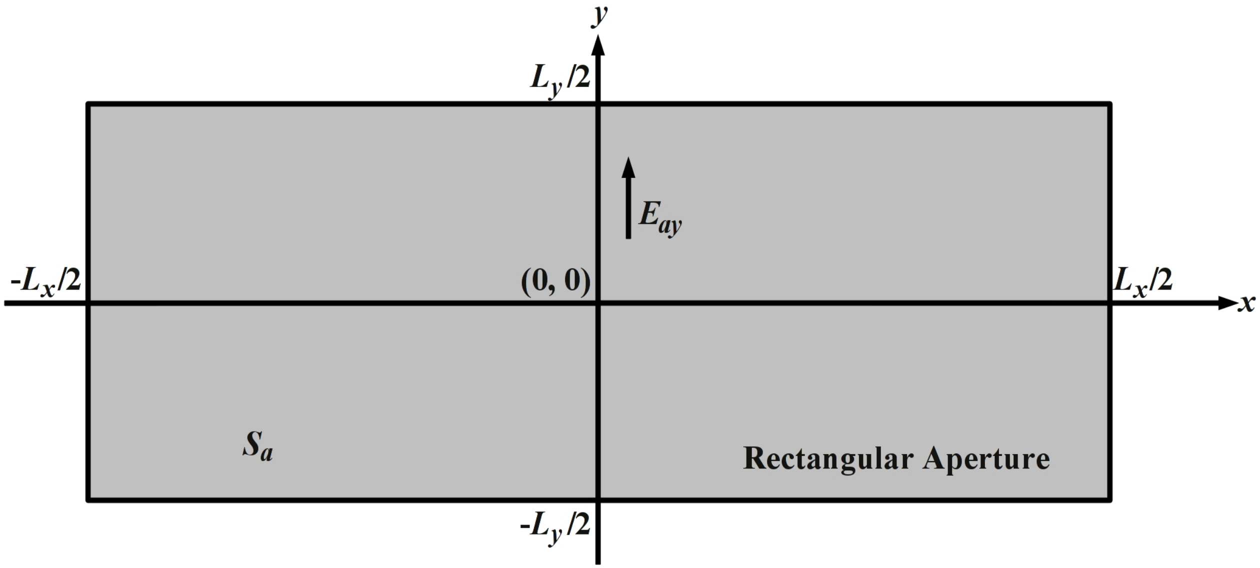


$$E_\theta = \left(j\beta \frac{e^{-j\beta r}}{2\pi r} \sin\phi \right) E_{ay} \iint_{S_a} e^{j\beta(\sin\theta\ \cos\phi)x} e^{j\beta(\sin\theta\ \sin\phi)y} dx\, dy$$

$$E_\phi = \left(j\beta \frac{e^{-j\beta r}}{2\pi r} \cos\theta\ \cos\phi \right) E_{ay} \iint_{S_a} e^{j\beta(\sin\theta\ \cos\phi)x} e^{j\beta(\sin\theta\ \sin\phi)y} dx\, dy$$

Uniform Rect. Aperture and Far Field - I

Assume that the aperture plane behaves like a perfect magnetic conductor. Let electric field E_{ay} be directed along the y-axis.



Using the general equations with limits of integration set to aperture size we obtain E_θ as:-

$$E_\theta = \left[\frac{j\beta \sin\phi \, e^{-j\beta r}}{2\pi r}\right] E_{ay} \int_{-\frac{L_x}{2}}^{\frac{L_x}{2}} e^{j\beta(\sin\theta \, \cos\phi)x} dx \int_{-\frac{L_y}{2}}^{\frac{L_y}{2}} e^{j\beta(\sin\theta \, \sin\phi)y} dy$$

$$= E_{ay} \, L_x \, L_y \left[j\beta \left(\frac{e^{-j\beta r}}{2\pi r}\right) \sin\phi\right] \left(\frac{\sin\left[\left(\beta L_x/2\right) u\right]}{\left(\beta L_x/2\right) u}\right) \left(\frac{\sin\left[\left(\beta L_y/2\right) v\right]}{\left(\beta L_y/2\right) v}\right)$$

where, $u = \sin\theta \cos\varphi$, and $v = \sin\theta \sin\varphi$

Uniform Rect. Aperture and Far Field - II

Similarly we obtain the E_φ component as shown below:

$$E_\phi = \left[\frac{j\beta\, e^{-j\beta r}\ \cos\theta\ \cos\phi}{2\pi r}\right] E_{ay} \int_{-\frac{L_x}{2}}^{\frac{L_x}{2}} e^{j\beta(\sin\theta\ \cos\phi)x} dx \int_{-\frac{L_y}{2}}^{\frac{L_y}{2}} e^{j\beta(\sin\theta\ \sin\phi)y} dy$$

$$= E_{ay}\, L_x\, L_y \left[j\beta \left(\frac{e^{-j\beta r}}{2\pi r}\right)\cos\theta\cos\phi\right]\left(\frac{\sin\left[(\beta L_x/2)\,u\right]}{(\beta L_x/2)\,u}\right)\left(\frac{\sin\left[(\beta L_y/2)\,v\right]}{(\beta L_y/2)\,v}\right)$$

where, $u = \sin\theta\cos\varphi$, and $v = \sin\theta\sin\varphi$

These sinc functions generates side lobes. The peaks of E_θ and E_φ depend on $L_x\, L_y$ and is directed along z-axis ($\theta = 0$). At $\varphi = 0, \pi$, $\sin\varphi = 0$. Similarly, at $\varphi = \pi/2, 3\pi/2$, $\cos\varphi = 0$. E_θ and E_φ vanish at these places. This is because E_{ay} is along y-axis. As $L_x >> \lambda$, the main beam narrows in x-z plane. As $L_y >> \lambda$, the main beam narrows in y-z plane.

Uniform Rect. Aperture and Far Field - III

The sinφ and cosφ appear in E_θ and E_φ since electric field is polarized. For unpolarized waves, these factors can be unity. Thus the normalized beam pattern P_n can be expressed as:

$$P_n(\theta, \phi) = \left(\frac{\sin(k_x u)}{k_x u}\right)^2 \left(\frac{\sin(k_y v)}{k_y v}\right)^2$$

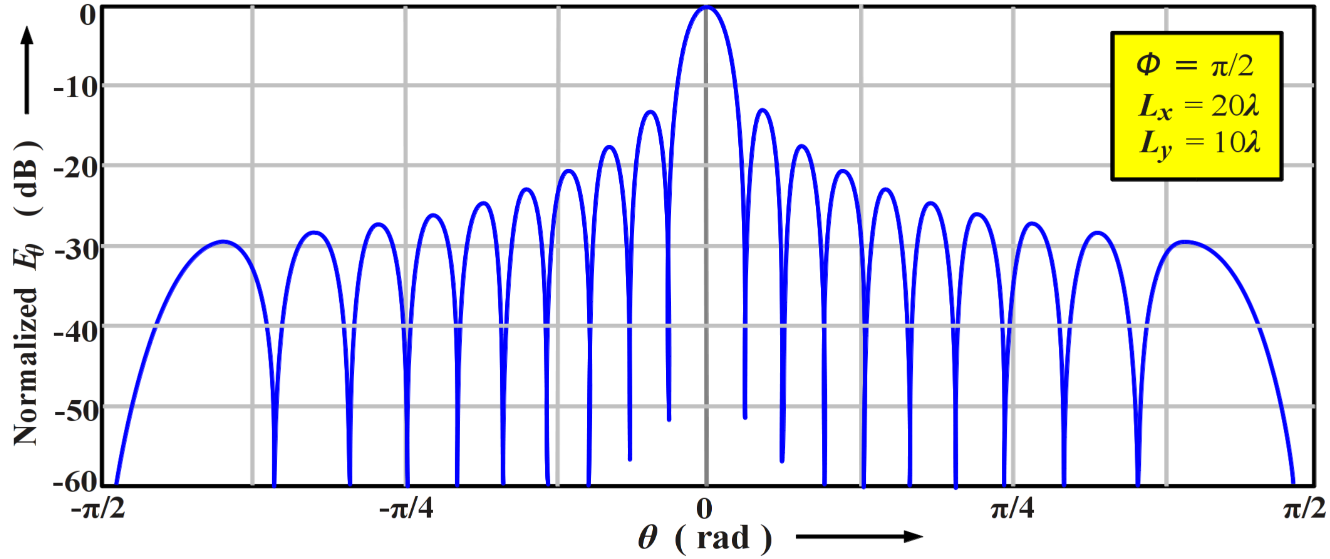
where, $$k_x = \frac{\beta L_x}{2} = \pi\left(\frac{L_x}{\lambda}\right), \; k_y = \frac{\beta L_y}{2} = \pi\left(\frac{L_y}{\lambda}\right)$$

Beam solid angle $$\Omega_A = \int_0^{2\pi}\int_0^{\pi} P_n(\theta, \phi)\, d\Omega = \frac{\lambda^2}{L_x L_y}$$

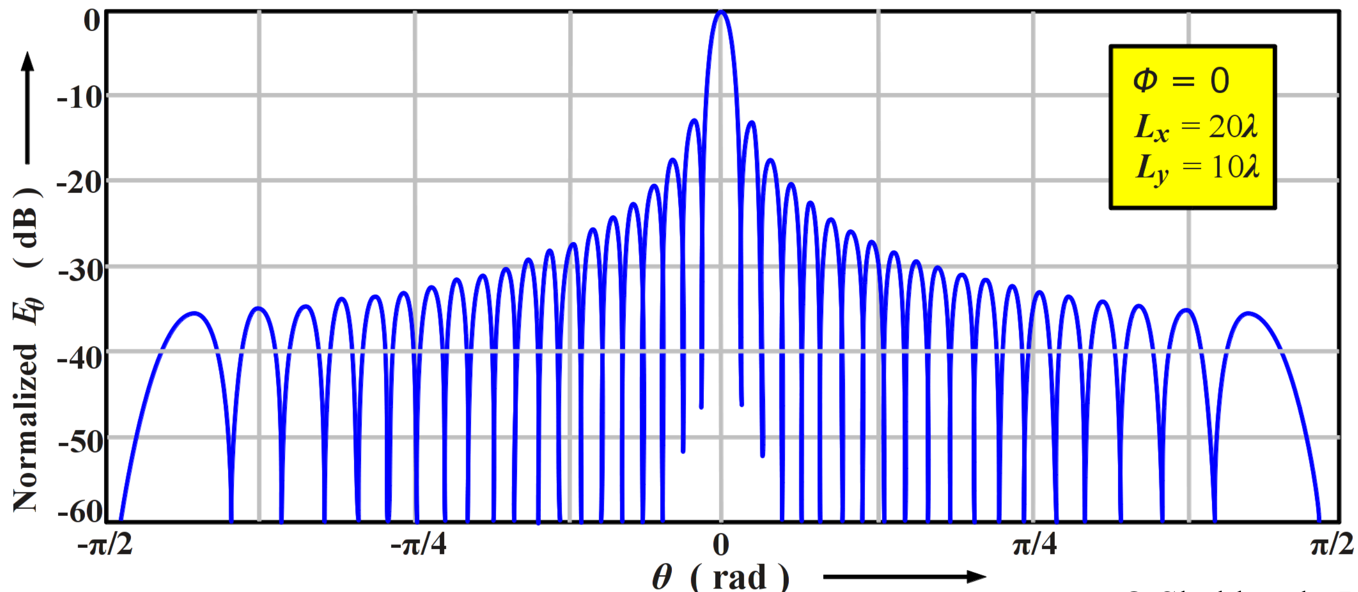
Directivity $$G_D = \frac{4\pi}{\Omega_A} = \left(\frac{4\pi}{\lambda^2}\right)(L_x L_y)$$

Uniform Rect. Aperture and Far Field - IV

Plot of E_θ in the y-z plane.

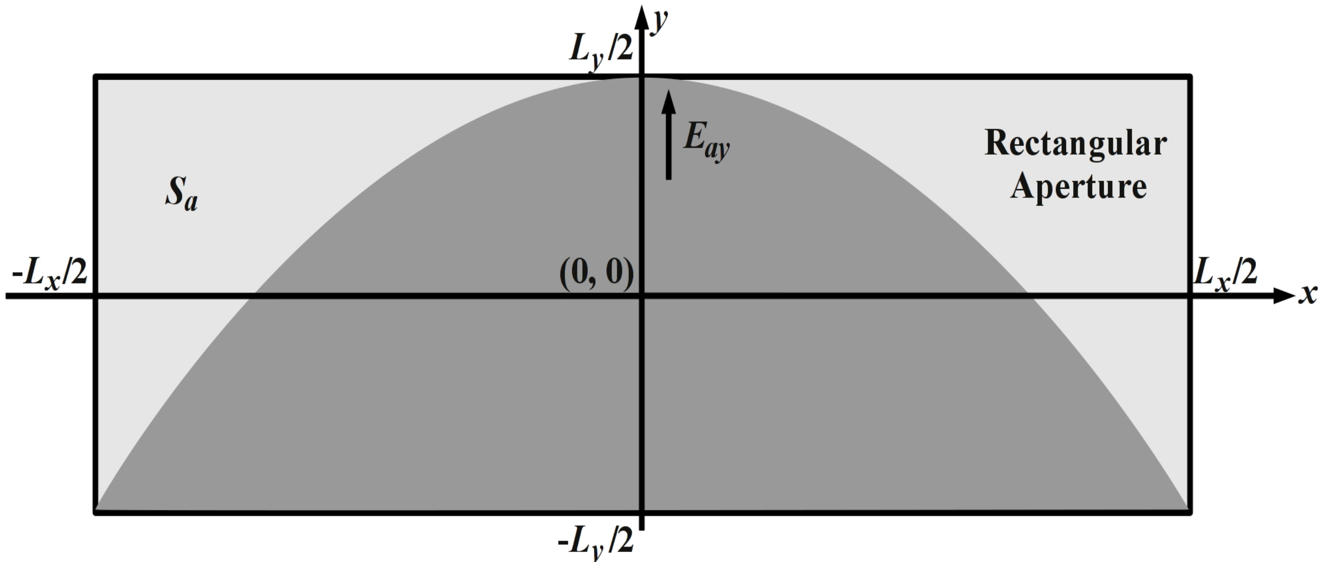


Plot of E_θ in the x-z plane.



Tapered Rect. Aperture and Far Field - I

Consider a rectangular aperture of dimension $L_x L_y$ with tapered variation of electric field E_{ay} directed along the y-axis.



The electric field variation across x-axis is given as:

$$E_{ay} = E_0 \cos\left(\frac{\beta\lambda}{4L_x}x\right) = \frac{1}{2}\left[e^{j\beta x(\lambda/2L_x)} + e^{-j\beta x(\lambda/2L_x)}\right]$$

where, E_0 is the peak intensity.

These kind of electric field variations can be observed in open ended waveguides operating in the TE_{10} mode. At $x = 0$, the electric field intensity is maximum, and it is minimum at $x = L_x/2$.

Tapered Rect. Aperture and Far Field - II

The far electric field functions E_θ and E_φ are expressed as:

$$E_\theta = \frac{E_0}{2} L_x L_y \left(j\beta \frac{e^{-j\beta r}}{2\pi r} \sin\phi \right) \times \left(\frac{\sin\left[(\beta L_x/2)\, u_1\right]}{(\beta L_x/2)\, u_1} + \frac{\sin\left[(\beta L_x/2)\, u_2\right]}{(\beta L_x/2)\, u_2} \right) \left(\frac{\sin\left[(\beta L_y/2)\, v\right]}{(\beta L_y/2)\, v} \right)$$

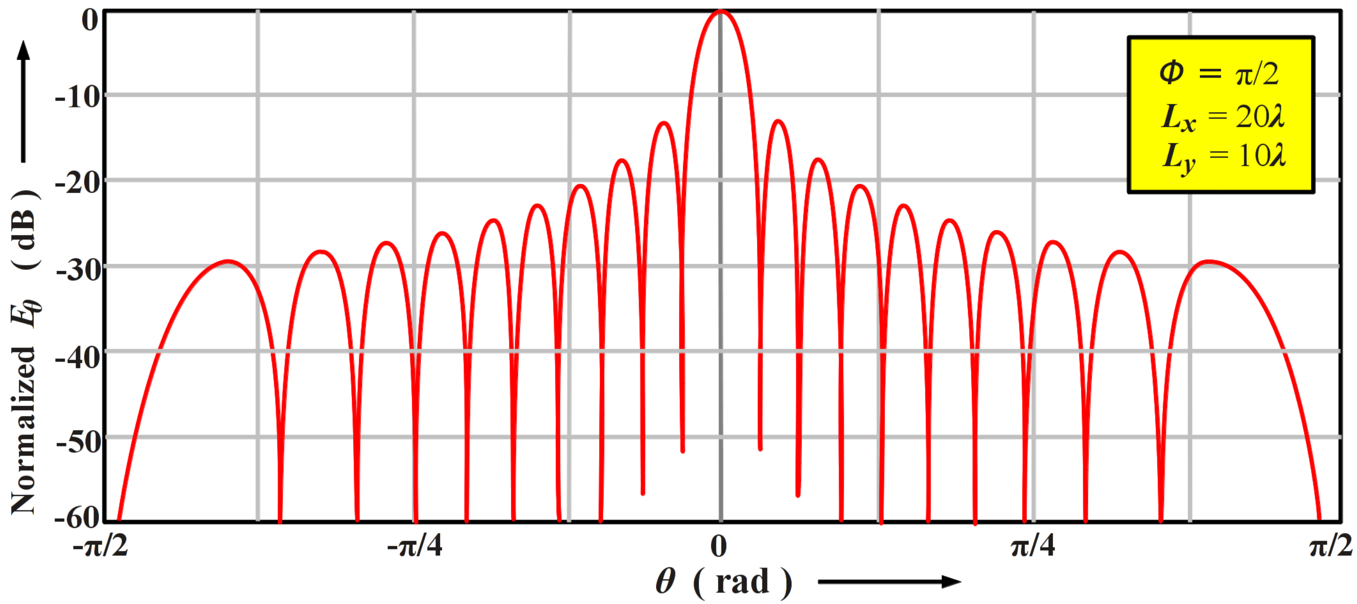
$$E_\phi = \frac{E_0}{2} L_x L_y \left(j\beta \frac{e^{-j\beta r}}{2\pi r} \cos\theta \cos\phi \right) \times \left(\frac{\sin\left[(\beta L_x/2)\, u_1\right]}{(\beta L_x/2)\, u_1} + \frac{\sin\left[(\beta L_x/2)\, u_2\right]}{(\beta L_x/2)\, u_2} \right) \left(\frac{\sin\left[(\beta L_y/2)\, v\right]}{(\beta L_y/2)\, v} \right)$$

where, $u_1 = \sin\theta\cos\phi + \lambda/(4L_x)$,

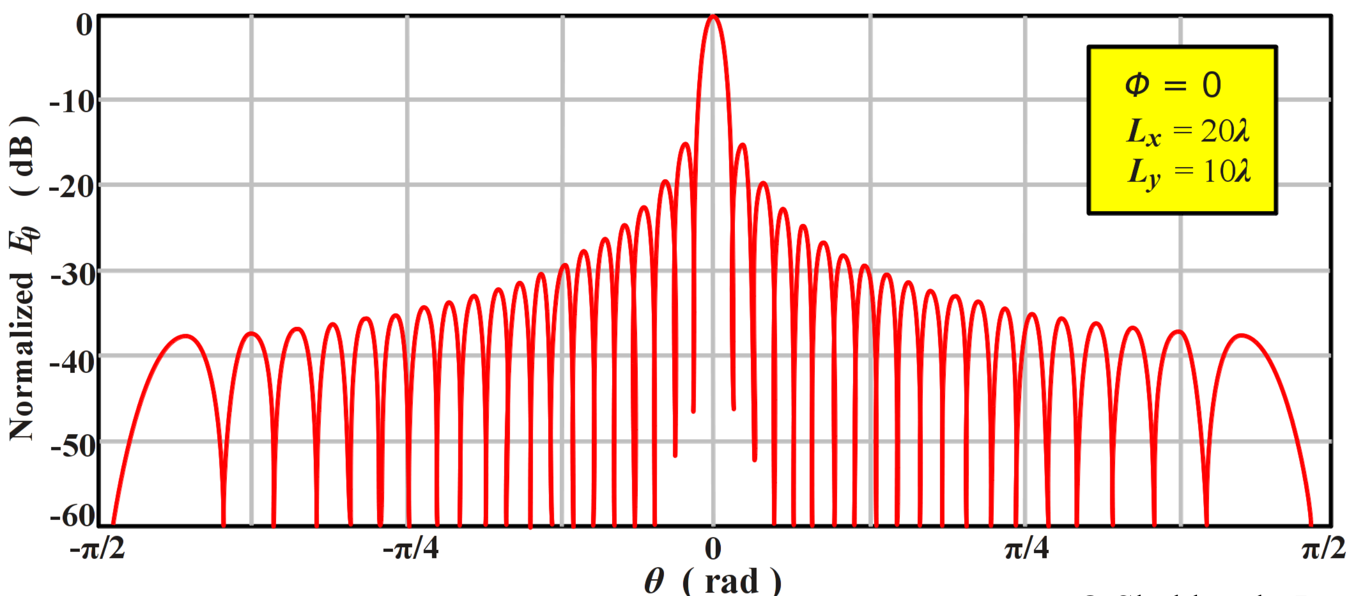
and $u_2 = \sin\theta\cos\phi - \lambda/(4L_x)$ and $v = \sin\theta\sin\phi$

Tapered Rect. Aperture and Far Field - III

Plot of E_θ in the *y*-*z* plane.

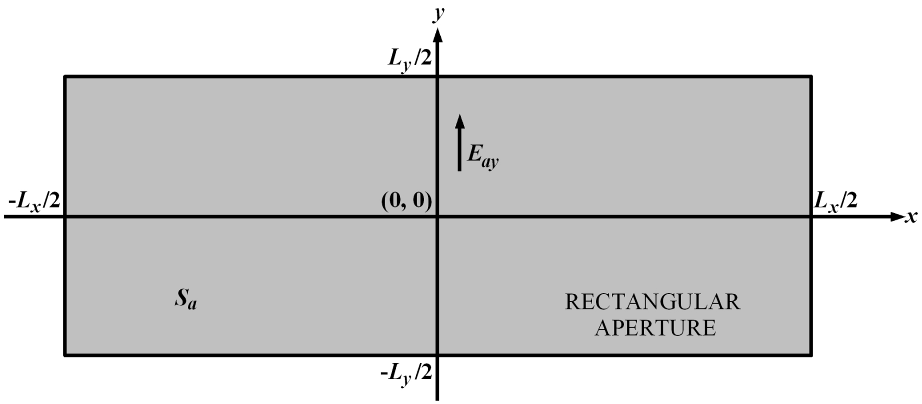


Plot of E_θ in the *x*-*z* plane.

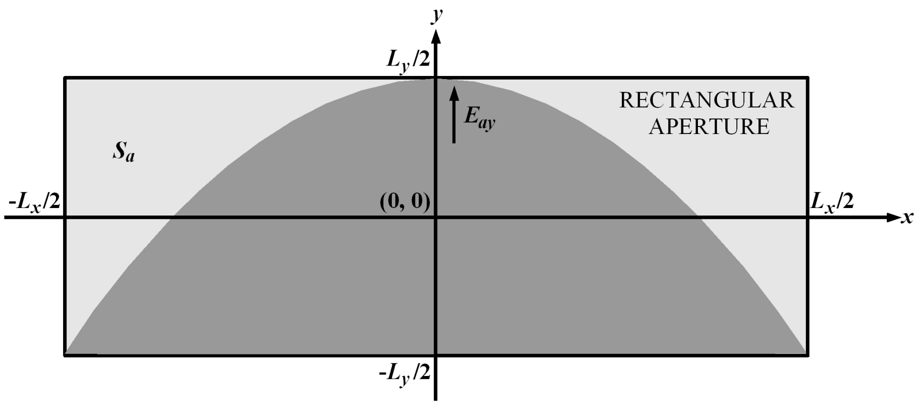


Taper vs. Uni. Illuminations: Far Fields - I

Comparison of near electric fields on the antenna aperture for (*i*) uniform illumination, and (*ii*) cosine tapered illumination.



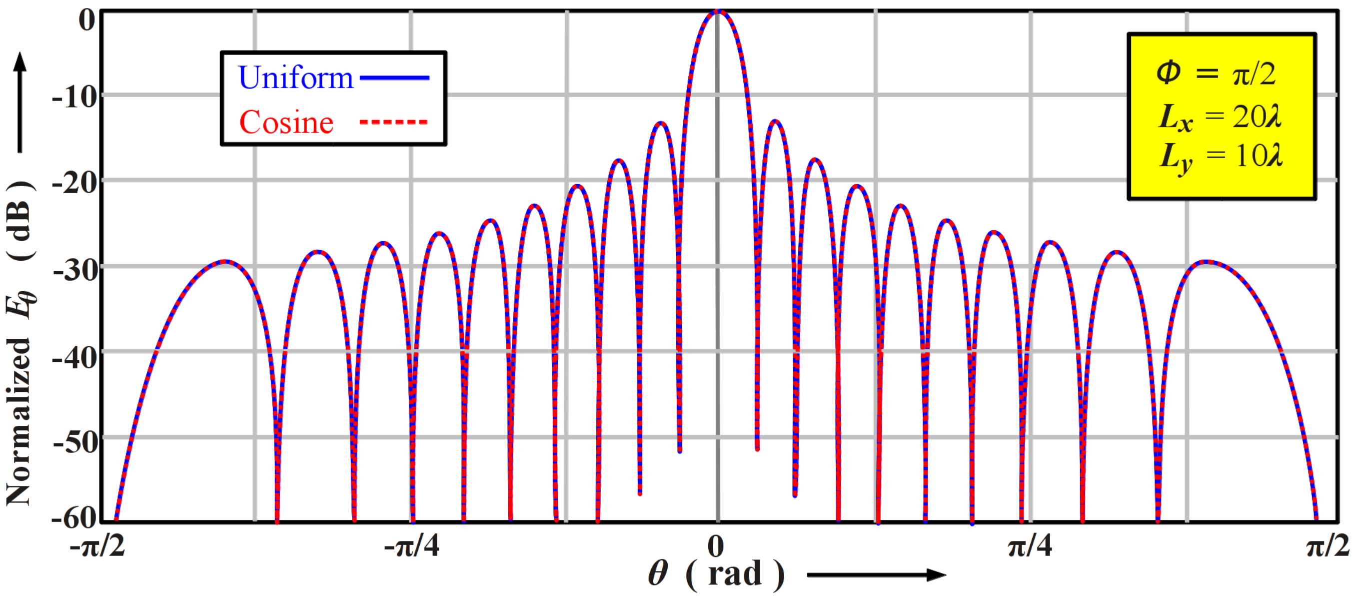
Uniform illumination



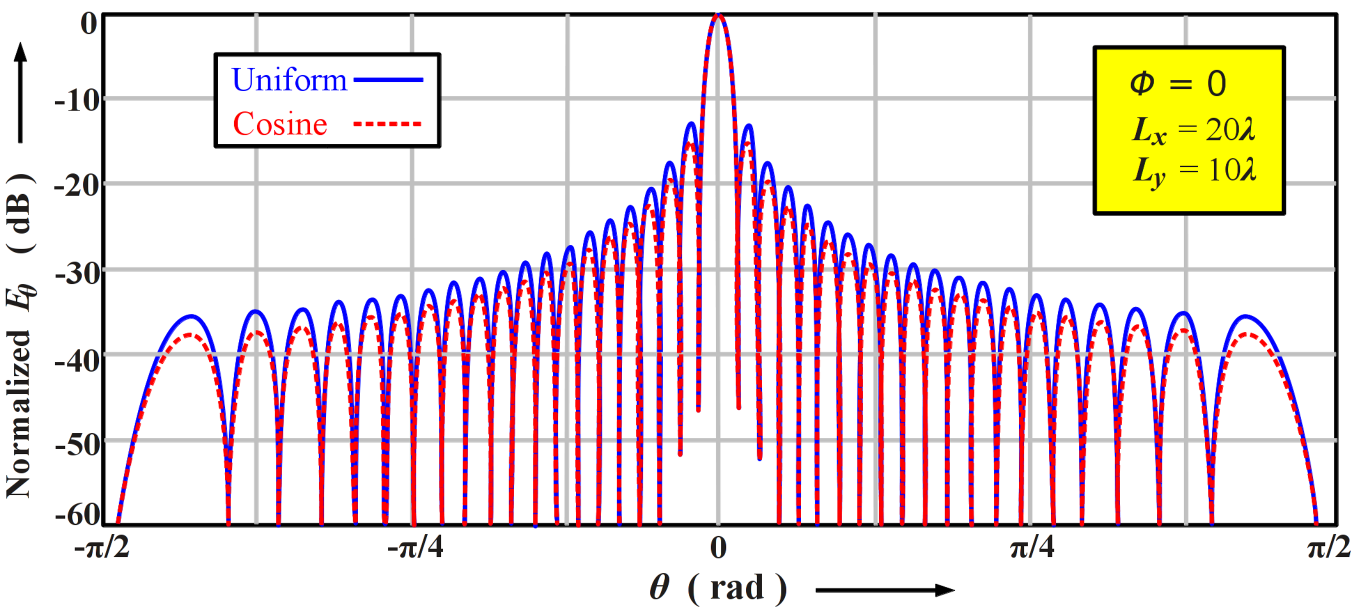
Cosine tapered illumination

Taper vs. Uni. Illuminations: Far Fields - II

Far field plot of E_θ in ***y-z*** plane for (*i*) uniform illumination, and (*ii*) cosine tapered illumination.



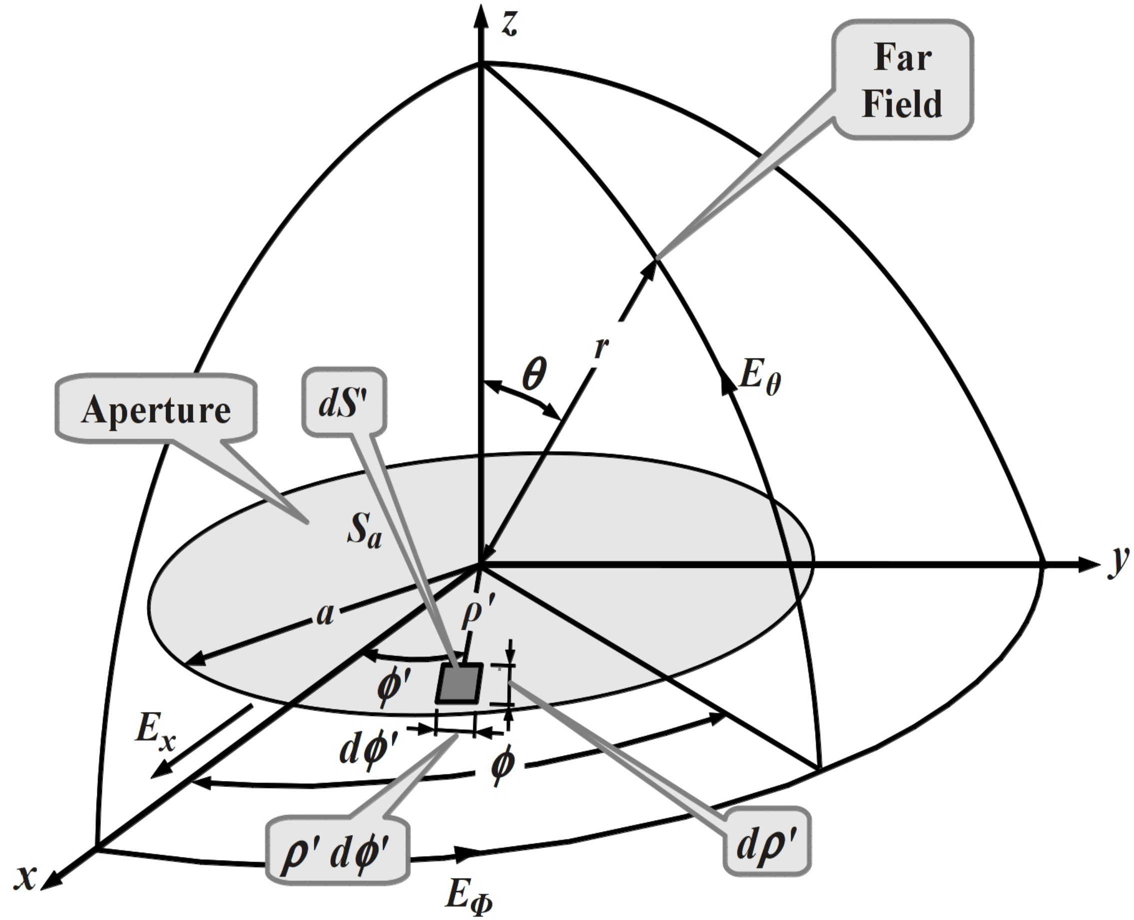
Far field plot of E_θ in ***x-z*** plane for (*i*) uniform illumination, and (*ii*) cosine tapered illumination.



Uniform Circular Aperture and Far Field - I

A circular aperture of radius a centered in a spherical coordinate system and placed in the x-y plane is shown.

Let electric field E_x be directed along the x-axis. Let dS' be infinitesimal area on the aperture at a distance ρ' subtending an angle φ'.



Infinitesimal area on aperture is $dS' = \rho\, d\phi'\; d\rho'$

Projection of ρ' towards r is $r' = \rho'\;\cos(\phi - \phi')\;\cos(\pi/2 - \theta) = \rho'\;\cos(\phi - \phi')\;\sin\theta$

Uniform Circular Aperture and Far Field - II

Components of far electric fields are:

$$E_\theta = \left(j\beta \frac{e^{-j\beta r}}{2\pi r} \cos\phi\right) \iint_{S_a} E_x \, e^{j\beta r'} dS'$$

$$= \left(j\beta \frac{e^{-j\beta r}}{2\pi r} \cos\phi\right) E_x \times \int_0^a \int_0^{2\pi} e^{j\beta \sin\theta \cos\left(\phi - \phi'\right)\rho'} d\phi \, \rho' \, d\rho'$$

$$= E_x \left(j\beta \frac{e^{-j\beta r}}{2\pi r} \cos\phi\right) \left(\frac{2J_1(\beta a \sin\theta)}{\beta a \sin\theta}\right)$$

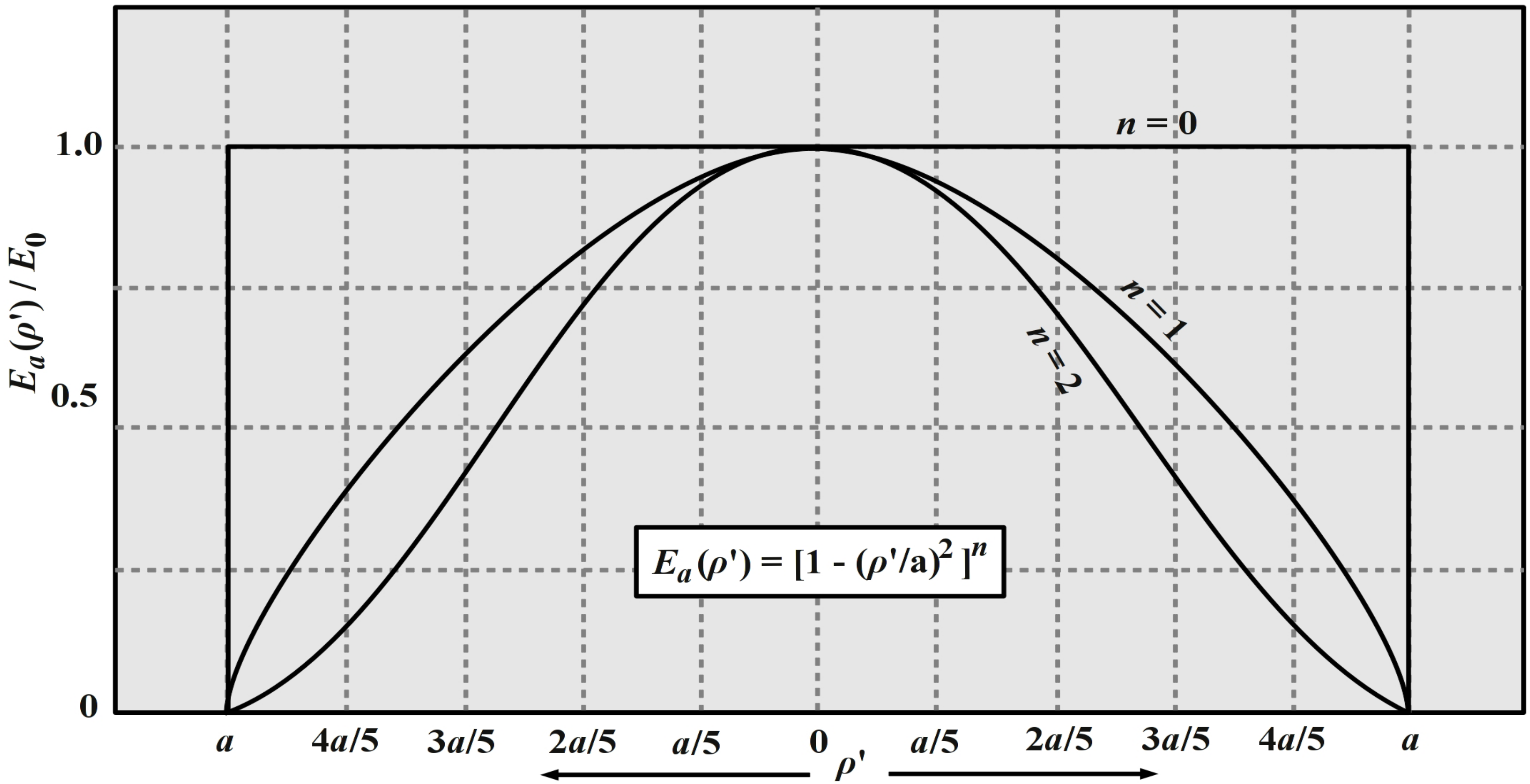
$$E_\phi = \left(j\beta \frac{e^{-j\beta r}}{2\pi r} \cos\theta \sin\phi\right) \iint_{S_a} E_x \, e^{j\beta r'} dS'$$

$$= \left(j\beta \frac{e^{-j\beta r}}{2\pi r} \cos\theta \sin\phi\right) E_x \int_0^a \int_0^{2\pi} e^{j\beta \sin\theta \cos\left(\phi - \phi'\right)\rho'} d\phi \, \rho' \, d\rho'$$

$$= E_x \left(j\beta \frac{e^{-j\beta r}}{2\pi r} \cos\theta \sin\phi\right) \left(\frac{2J_1(\beta a \sin\theta)}{\beta a \sin\theta}\right)$$

Since the above are as a result of circular aperture excitation, the far field variations are guided by a Bessel function of first kind.

Tapered Circular Aperture and Far Field - I



Let the electric field peaks at origin of the aperture and falls as moved away in the *x-y* plane. Variation of the electric field is assumed parabolic function of ρ'.

$$E_a(\rho') = E_0\left[1-\left(\frac{\rho'}{a}\right)^2\right]^n \quad \text{where, } n = 0,\ 1,\ 2,\ 3\ldots$$

Tapered Circular Aperture and Far Field - II

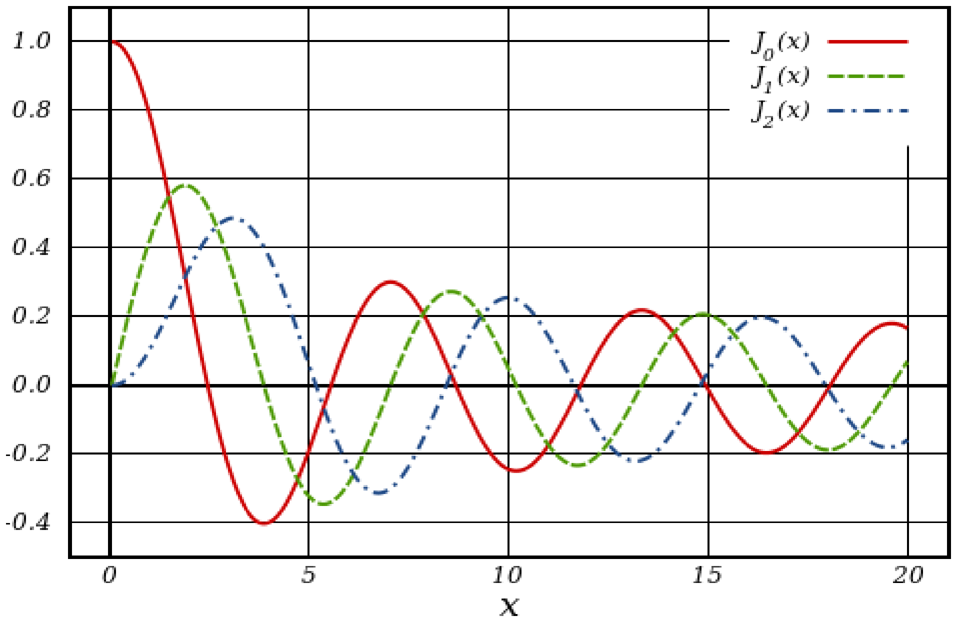
Components of far electric fields are:

$$E_\theta = \left(j\beta\frac{e^{-j\beta r}}{2\pi r}\cos\phi\right)\iint_{S_a} E_0\left[1-\left(\frac{\rho'}{a}\right)^2\right]^n \; e^{j\beta r'}dS'$$

$$= E_0\left(j\beta\frac{e^{-j\beta r}}{2\pi r}\cos\phi\right)\left(\frac{\pi a^2}{n+1}\right)\left(\frac{2^{n+1}(n+1)!J_{n+1}(\beta a\sin\theta)}{(\beta a\sin\theta)^{n+1}}\right)$$

$$E_\phi = \left(j\beta\frac{e^{-j\beta r}}{2\pi r}\cos\theta\,\sin\phi\right)\iint_{S_a} E_0\left[1-\left(\frac{\rho'}{a}\right)^2\right]^n \; e^{j\beta r'}dS'$$

$$= \left(j\beta\frac{e^{-j\beta r}}{2\pi r}\cos\theta\,\sin\phi\right)\left(\frac{\pi a^2}{n+1}\right)\left(\frac{2^{n+1}(n+1)!J_{n+1}(\beta a\sin\theta)}{(\beta a\sin\theta)^{n+1}}\right)$$



In above, we find that for any parabolic excitation on a circular aperture results in a far field variation guided by a Bessel function of first kind.

General Circular Aperture Far Fields - I

The aperture field is related to the far field using a Fourier Transform relationship.

If we assume the aperture illumination to be un-polarized, then this relationship between the near electric field E_{Near} and the far electric field E_{Far} are expressed as:

$$E_{\text{Far}}\,(\text{direction cosines}) = K_A\;\mathcal{F}\left[E_{\text{Near}}(\text{rect. coords.})\right]$$

$$E_{\text{Far}}(l,m) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_{\text{Near}}\,(u,v)\;e^{-j2\pi l u}e^{-j2\pi m\,v}du\,dv$$

General Circular Aperture Far Fields - II

$$E_0 = 1,\ a = 10\lambda,\ E(\rho) = 0 \text{ for } \rho > a,\ \text{where } \rho = \sqrt{u^2 + v^2}$$

Near Fields

Cone

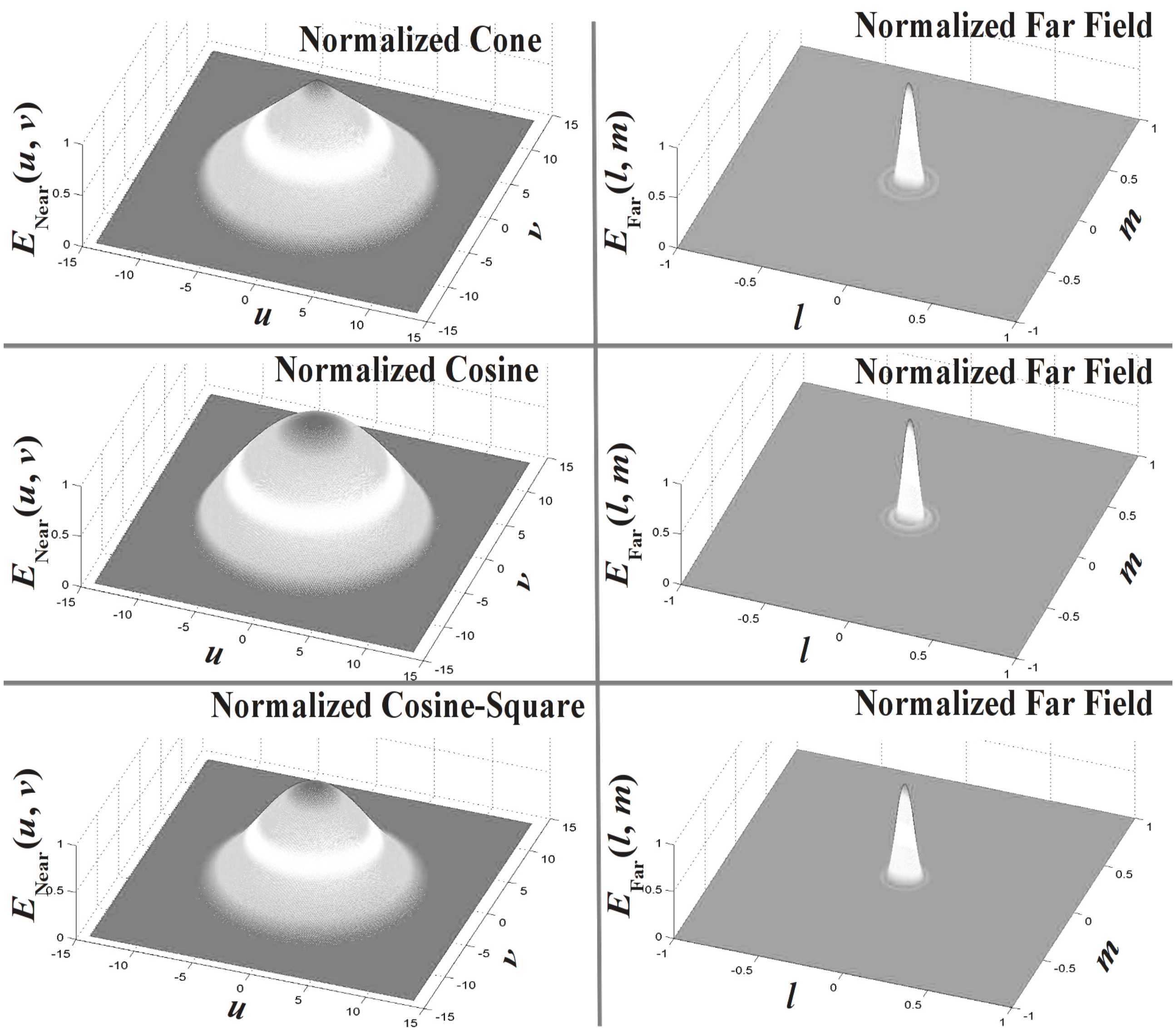
$$E(\rho) = E_0 \left(\frac{a - \rho}{a} \right)$$

Cosine

$$E(\rho) = E_0 \cos\left(\frac{\pi}{2a} \rho \right)$$

Cosine Square

$$E(\rho) = E_0 \left[\cos\left(\frac{\pi}{2a} \rho \right) \right]^2$$



General Circular Aperture Far Fields - III

$$E_0 = 1,\ a = 10\lambda,\ E(\rho) = 0 \text{ for } \rho > a,\ \text{where}\ \rho = \sqrt{u^2 + v^2}$$

Near Fields

Gaussian

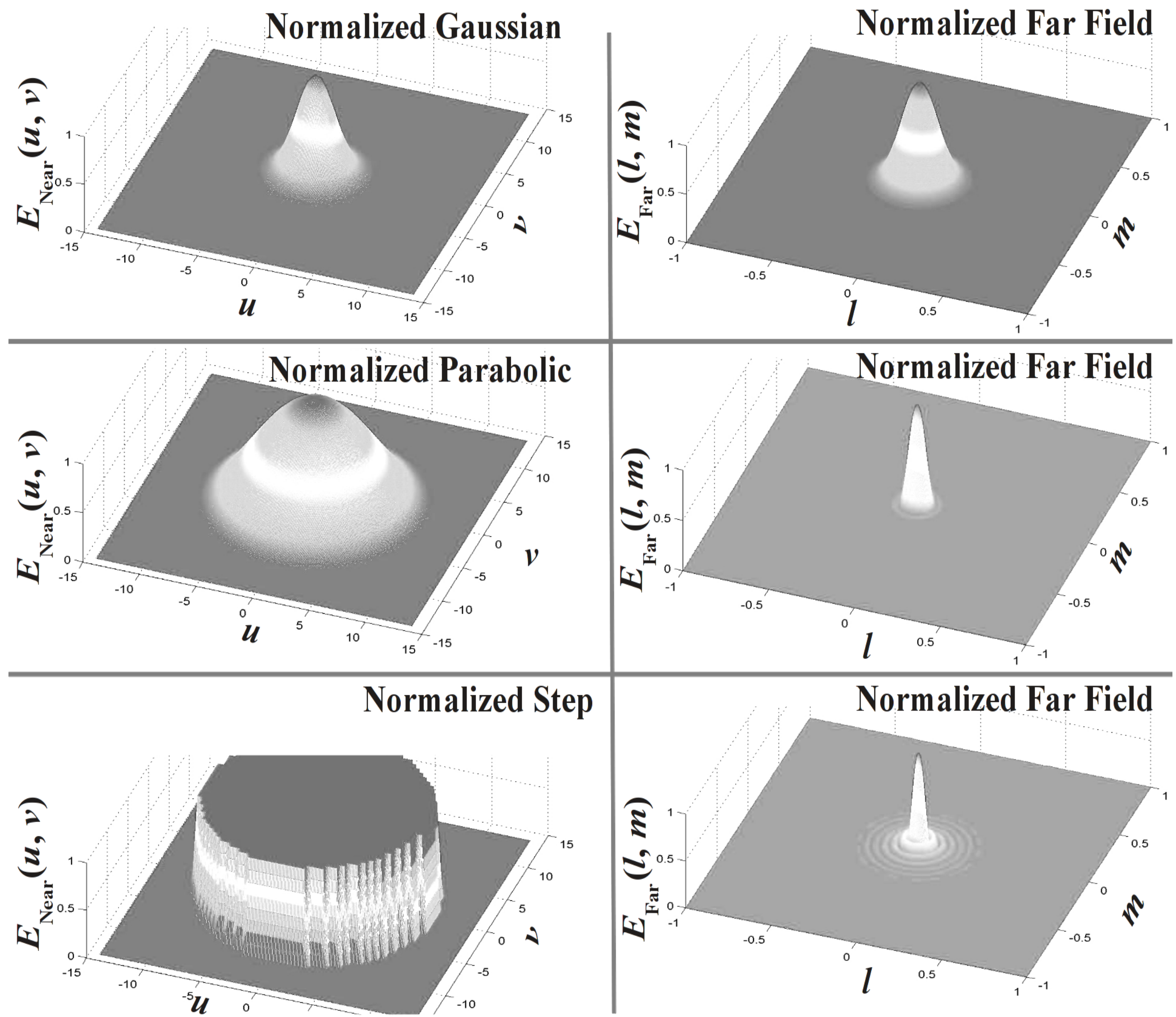
$$E(\rho) = E_0 \exp\left(-\frac{\rho^2}{k^2}\right)$$

Parabolic

$$E(\rho) = E_0 \left(1 - \frac{\rho^2}{a^2}\right)^n$$

Step

$$E(\rho) = E_0$$



Radio Arrays

Generally two types of radio arrays are used in radio astronomy: (*i*) phased array, and (*ii*) correlator array.

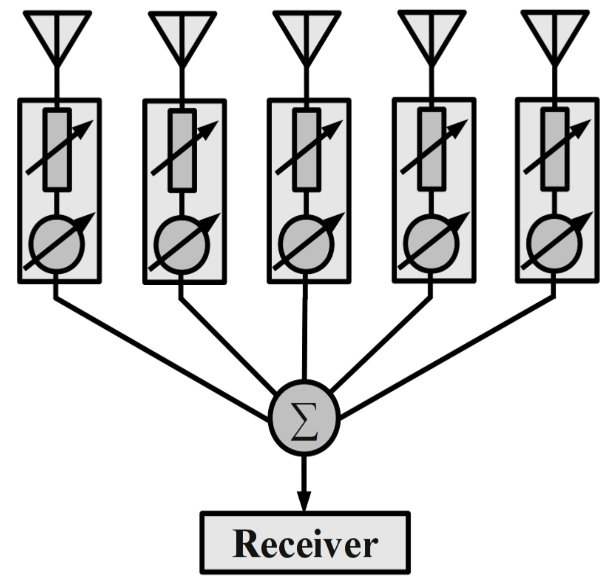
Advantages of arrays

- Difficulties in constructing a large single dish can be overcome by arrays.
- Multiple increase in signal to noise ratio as compared to a single dish.
- Increases overall directivity and resolution of the telescope.
- Effect of Earth's rotation can aid to super-synthesis.

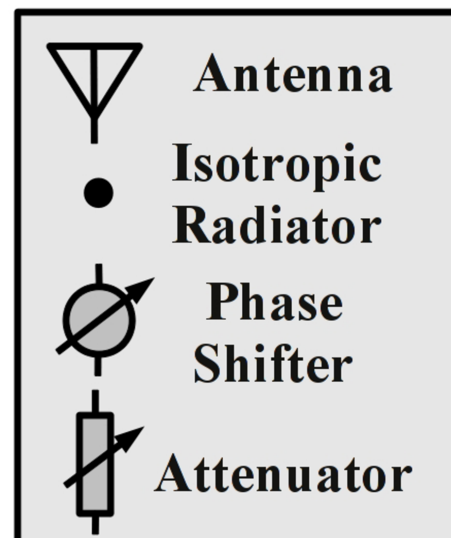
Note: The GMRT configures itself as correlator array when observing spectral lines and continuum radio sources. It configures itself as a phased array when observing pulsars.

Linear Antenna Arrays - I

A linear array is shown, where all the antennas are placed in a line and equally spaced. The radiation pattern and other electrical characteristics of the antennas are identical. The signals received by each antenna are passed through a combination of attenuator and phase shifter. The signals are then summed and fed to the receiver.

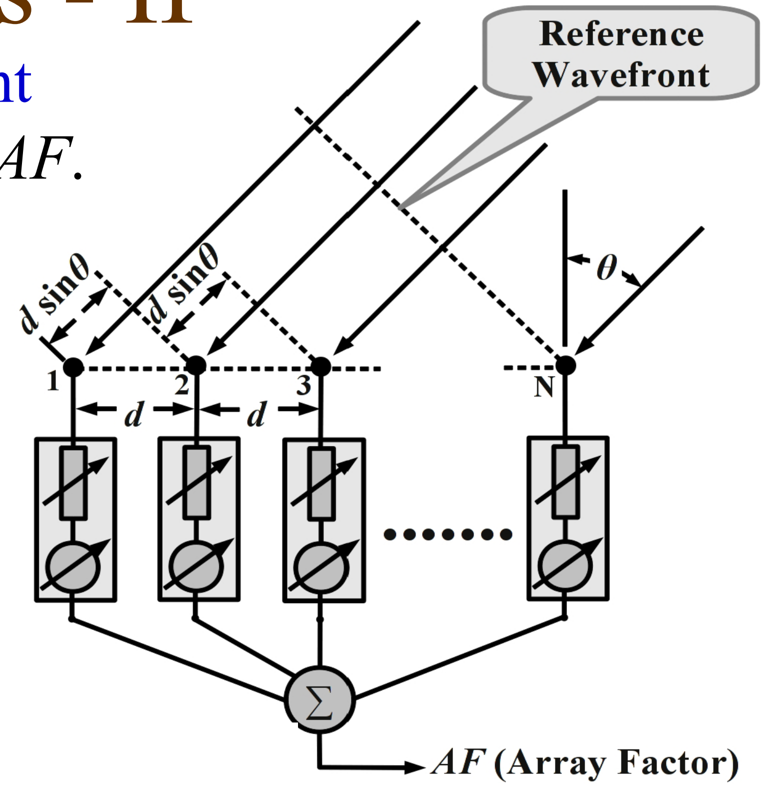


Beam pattern of an antenna array formed by non isotropic but similar point antennas is the product of the pattern of the individual antenna with pattern of an array formed by replacing all the antennas with isotropic antennas.



Linear Antenna Arrays - II

The pattern of the array with isotropic point antennas is also called the ***array factor*** or *AF*.



Analysis of an array is done in two steps:

(*i*) find the array factor *AF* by replacing the antennas with isotropic radiators, and then

(*ii*) multiply with the radiation pattern of an individual antenna.

N = No. of antennas.

Signal reaching the adder from N^{th} antenna is $A_N e^{j0} = A_N$

Similarly, signals reaching from $(N\text{-}1)^{th}$, $(N\text{-}2)^{th}$, …, 1^{st} antennas are:

$$A_{N-1}e^{j\beta d\sin\theta},\ A_{N-2}e^{j2\beta d\sin\theta},\ \ldots,\ A_1 e^{j(N-1)\,\beta d\sin\theta}$$

Since all antennas are identical, the magnitude contributions are same:

$$|A_n| = A_1 \quad \text{for } n = 1 \text{ to } N$$

Linear Antenna Arrays - III

In general, the signal from the n^{th} antenna is given as:

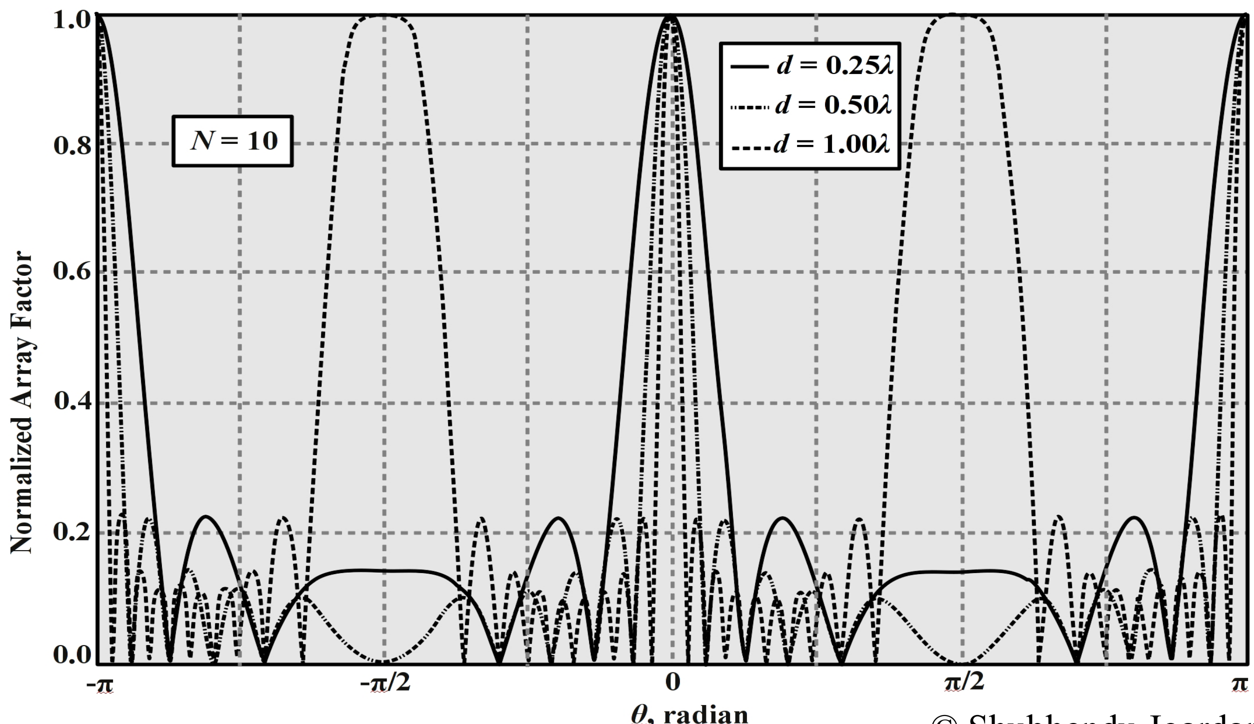
$$A_n = A_1 e^{j(N-n)\,\beta d\cos\theta} = A_1 e^{j(N-n)\,\psi} \quad \text{where, } \psi = \beta d \sin\theta \text{ and } \beta = \frac{2\pi}{\lambda}$$

The output of the adder is the array factor K_{AF} given as:

$$K_{AF}(\theta) = A_1 \sum_{n=1}^{N} e^{j(N-n)\,\psi} = A_1 \frac{1 - e^{jN\psi}}{1 - e^{j\psi}} = A_1\, e^{j(N-1)\,\psi/2} \left[\frac{\sin(N\psi/2)}{\sin(\psi/2)}\right]$$

Note: Here, applied attenuation and phase shifts are zero.

Normalized variation of the array factor with respect to θ for different spacing between the isotropic radiators for $N = 10$. This type of response is sometimes called ***grating response***.



Linear Antenna Arrays - IV

Consider the case when the spacing is fixed, but different phase shifts and attenuations are applied to the individual isotropic radiator. Let the phase shifts applied to the n^{th} antenna be κ_n and the attenuation be α_n.

Then the array factor can be expressed as:

$$K_{AF}(\theta) = A_0 \sum_{n=1}^{N} \alpha_n e^{j[\beta(N-n)\,\psi + \kappa_n]}$$
$$\text{where, } 0 \leq \alpha_n \leq 1,\ \ 0 \leq \kappa_n \leq 2\pi$$

Here, A_0 is the largest amplitude of the signal out of all elements.

If the spacing between the array is not uniform, the array factor may be expressed as:

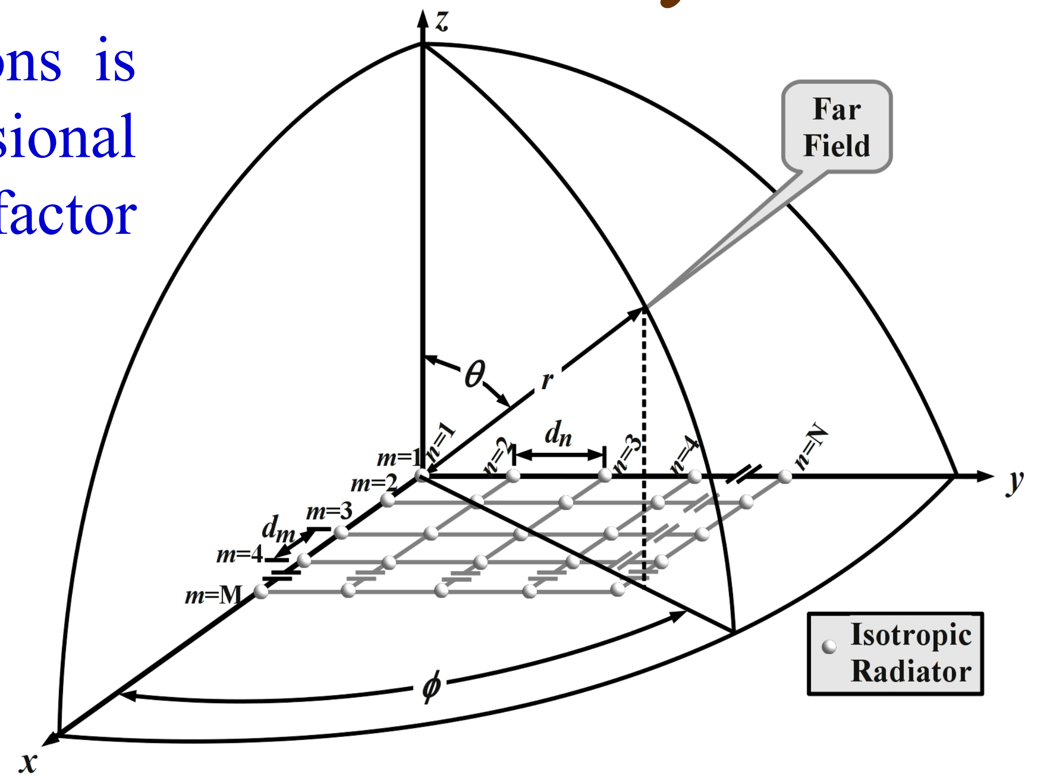
$$K_{AF}(\theta) = A_0 \sum_{n=1}^{N} \alpha_n e^{j[\beta(N-n)\,\psi_n + \kappa_n]}$$
$$\text{where, } 0 \leq \alpha_n \leq 1,\ \ 0 \leq \kappa_n \leq 2\pi,\ \ \psi_n = d_n \sin\theta$$

Here, d_n represents the distance between $(n+1)^{\text{th}}$ and n^{th} element.

By adjusting amplitude and phases of the elements of an array one can control the beam pattern in a plane.

Two Dimensional Antenna Arrays - I

Beam control in two dimensions is possible using a two dimensional array as shown. Here, the array factor *AF* is a function of both θ and φ.



Minimum spacing are:

(*i*) d_m along *x*-axis.

(*ii*) d_n along *y*-axis.

Maximum elements is *M* along *x*-axis and *N* along *y*-axis.

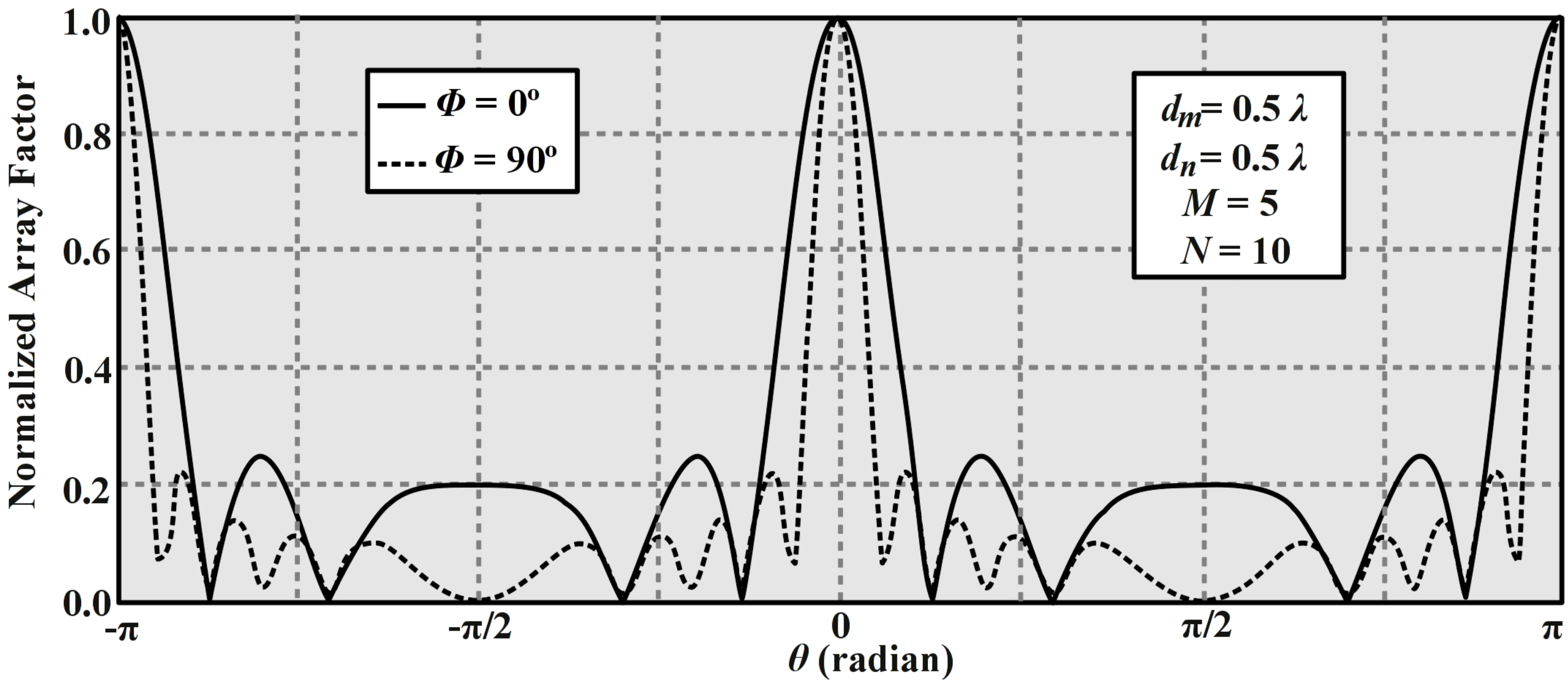
If no attenuation and or phase shift is applied the array factor is:

$$K_{AF}(\theta,\phi) = A_0 \sum_{m=1}^{\mathrm{M}} e^{j\beta(\mathrm{M}-m)\,\psi_m} \sum_{n=1}^{\mathrm{N}} e^{j\beta(\mathrm{N}-n)\,\psi_n}$$
$$\text{where, } \psi_m = d_m \sin\theta\cos\phi,\ \ \psi_n = d_n \sin\theta\sin\phi$$

If attenuation α_{mn} and phase shift κ_{mn} is applied, the array factor is:

$$K_{AF}(\theta,\phi) = A_0 \sum_{n=1}^{\mathrm{N}} \sum_{m=1}^{\mathrm{M}} \alpha_{mn}\, e^{\kappa_{mn}}\, e^{j\beta[(\mathrm{M}-m)\,\psi_m + (\mathrm{N}-n)\,\psi_n]}$$
$$\text{where, } 0 \le \alpha_{mn} \le 1,\ \ 0 \le \kappa_{mn} \le 2\pi$$

Two Dimensional Antenna Arrays - II

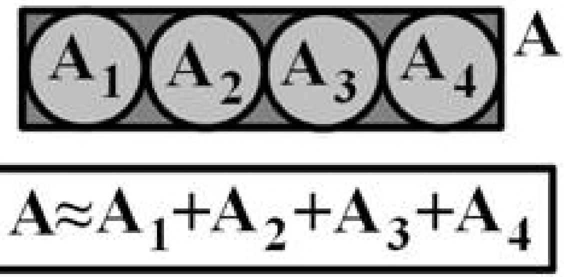


Normalized variation of the array factor with respect to θ in two orthogonal planes (y-z plane where φ = 90º, and x-z plane where φ = 0º) for different spacing between the isotropic radiators for M = 5, N = 10. This type of response is sometimes called grating response.

Radio Arrays (Filled Phased Array) - I

Synthesizing a large Rectangular Aperture using small Apertures

Consider four closely spaced fixed antennas pointed towards zenith. They approximately synthesize a rectangular aperture *A* for a radio waves arriving from a source at zenith.

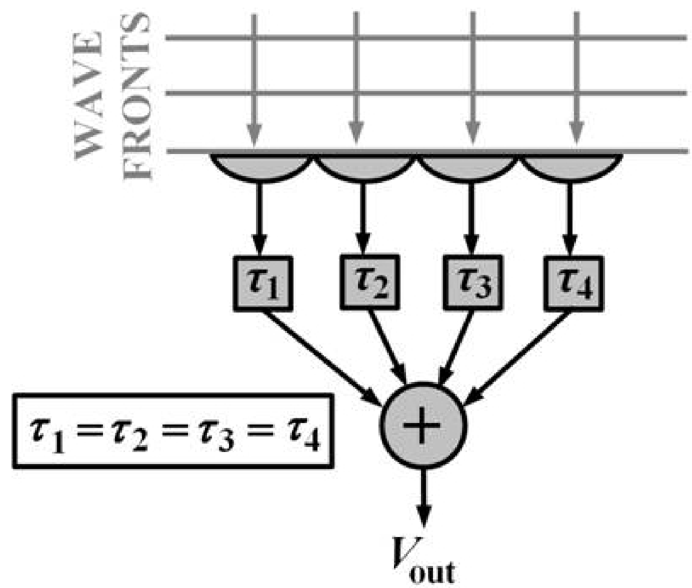


The synthesized aperture *A* can be expressed as the sum of individual apertures:

$$A \approx A_1 + A_2 + A_3 + A_4 = \sum_{1}^{4} A_i$$

At any instant of time, the same wave-front reaches all the individual apertures.

$$\tau_1 = \tau_2 = \tau_3 = \tau_4$$

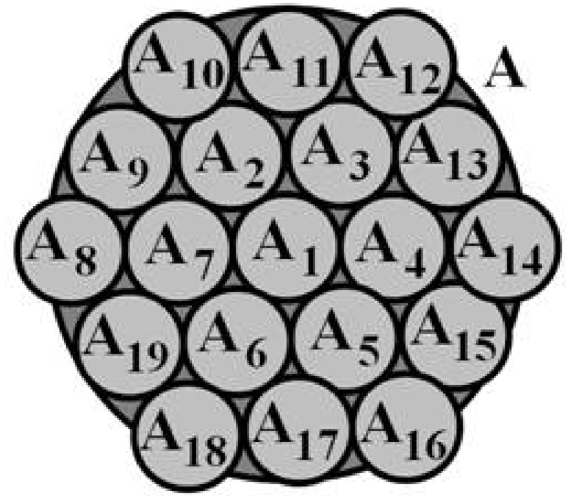


Disadvantage: The plane of the array must be parallel to the wave-fronts. Not suitable for a moving source on the sky.

Radio Arrays (Filled Phased Array) - II

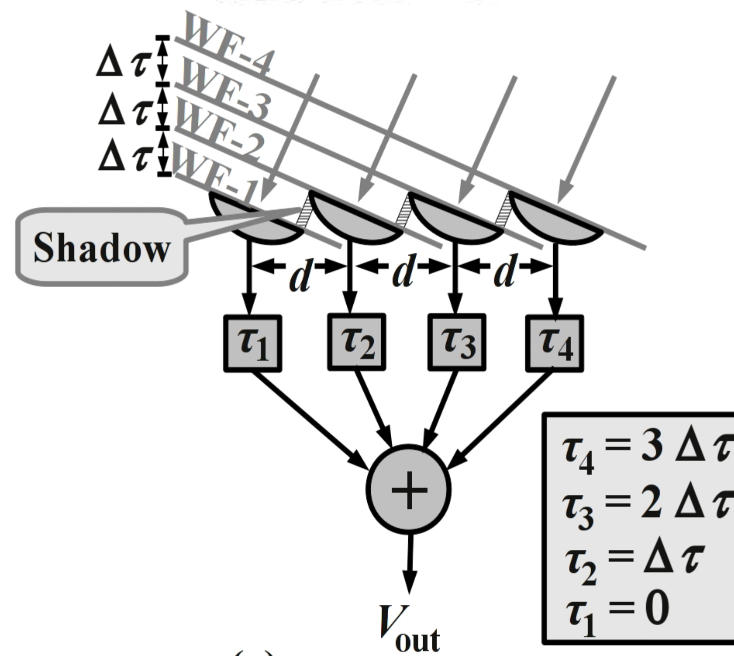
Synthesizing a large Circular Aperture using small Apertures

Consider many closely spaced fixed antennas pointed towards zenith as shown. They approximately synthesize a large circular aperture A for a radio waves arriving from a source at zenith.



Synthesized aperture $A \approx \sum_{1}^{19} A_i$

Tracking Antenna Array



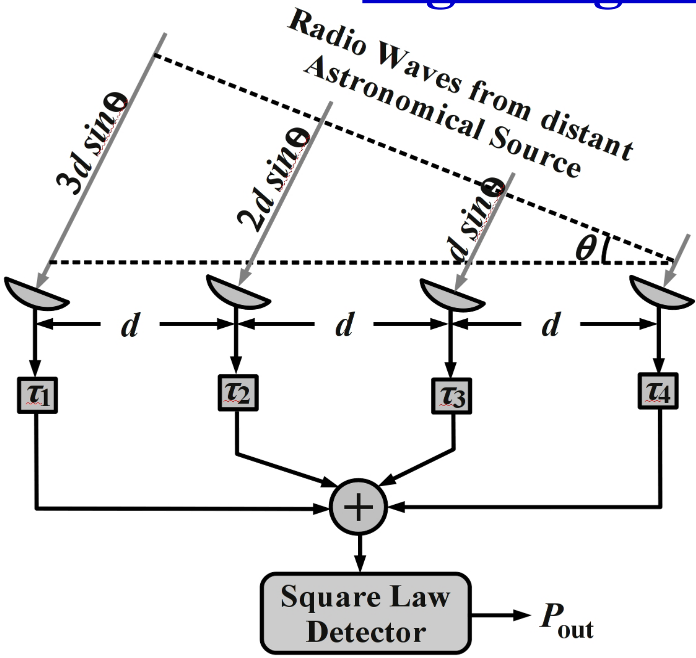
The disadvantages of a tracking array are:

(*i*) The effective area of the synthesized antenna decreases as the source moves to the horizon.

(*ii*) The delay has to constantly adjusted for a moving source

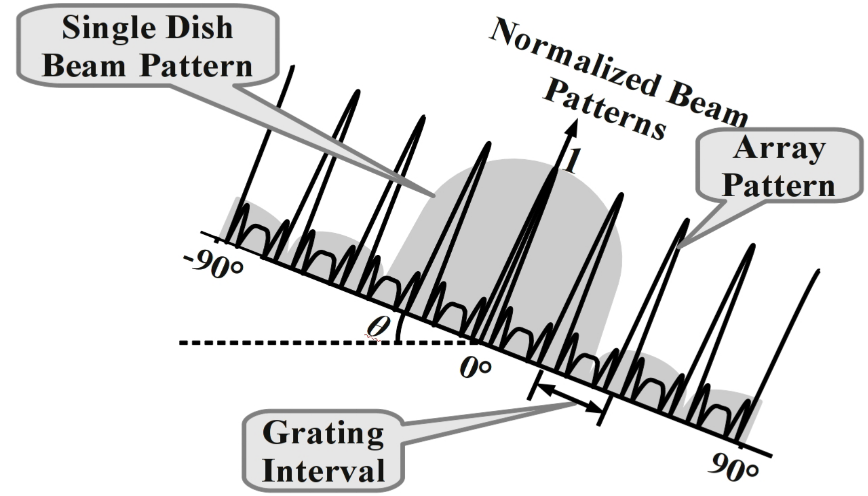
Radio Arrays (Grating Phased Array)

A grating array and its response



$\tau_1 = 0$ $\tau_2 = \Delta\tau$ $\tau_2 = 2\,\Delta\tau$ $\tau_4 = 3\,\Delta\tau$

where, $\Delta\tau = d\,\sin\theta / c$

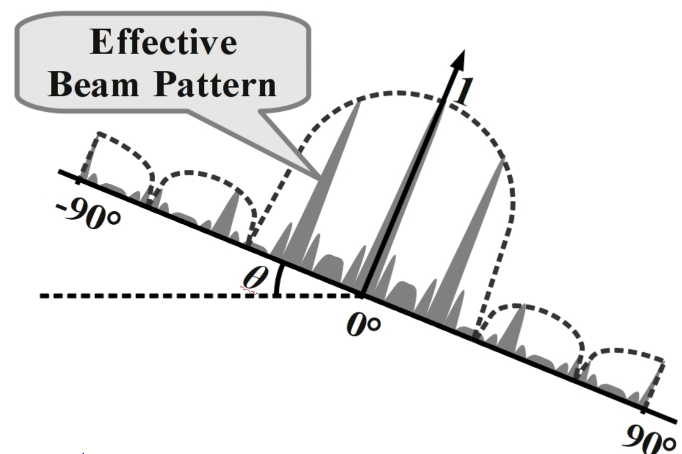


Normalized power patterns of a dish and an array of isotropic antennas.

$$P_{out} = K_s\,(V_1 + V_2 + V_3 + \ldots)^2 = K_s \left(\sum_{i=1}^{n_a} V_i\right)^2$$

K_s is proportionality constant.

Signals are added after phase correction using appropriate delays.



Effective system response.

Assignment Problems-I

1. A telescope has an objective of diameter of 2 m. If the wavelength is 500 nm, find its angular resolution.

Hint: Use

$$\alpha' = 1.22\frac{\lambda}{D} \approx \frac{\lambda}{D}$$

2. The dish diameter of an antenna is 45 meters. Find its angular resolution at the frequencies (*i*) 150 MHz (*ii*) 233 MHz, (*iii*) 327 MHz, (*iv*) 610 MHz and (*v*) 1420 MHz.

Hint: Calculate the wavelengths using $\lambda = c/\nu$ and use above equation.

3. Using a diagram describe the common antenna-feed mountings and focusing arrangements used in single dish radio telescopes.

4. Explain the terms: (*i*) spill-over, (*ii*) illumination leakage, (*iii*) side-lobe pick-up, and (*iv*) back-lobe pick-up. How do these parameters change with change in dish inclination from zenith? Explain using a diagram.

Assignment Problems-II

5. Explain the various efficiencies applicable to a dish antenna system: (*i*) antenna efficiency of antenna-feed, (*ii*) spill-over efficiency, (*iii*) dish leakage efficiency, (*iv*) surface smoothness efficiency, (*v*) illumination efficiency and (*vi*) polarization efficiency.

6. What is meant by aperture efficiency of a dish antenna system? Show its dependencies on other efficiencies mentioned in problem no. 5 using an expression.

Hint: $\eta_A = \eta_a \, \eta_{sp} \, \eta_{msh} \, \eta_{rms} \, \eta_{ill} \, \eta_{pol}$

7. How does the effective gain depend on aperture efficiency? Using an equation calculate the effective gain of a dish having a diameter of 45 meters and an aperture efficiency of 0.4 at a frequency of 1420 MHz.

Hint: Use

$$G_{eff} = \eta_A \left(\frac{\pi D}{\lambda} \right)^2$$

Assignment Problems-III

8. Under what conditions reduced aperture illumination is preferable? Tabulate its advantages and disadvantages.

9. Explain the difference between a non-planar and a planar log periodic antenna.

10. Explain the difference between log periodic dipole array and a log periodic antenna.

11. Design a log periodic dipole array having a gain of 7.5 dBi and a 1:4 bandwidth. The minimum frequency is 200 MHz.
Hint: Use the design graph and calculate τ and σ. Calculate the longest half-wave dipole using given frequency and proceed.

Assignment Problems-IV

12. Describe the differences between E-plane and H-plane sectoral horn antennas.
Hint: See the figure of horn antennas.

13. Describe the differences between pyramidal and conical horns.
Hint: See the figure of horn antennas.

14. What is the effective aperture area of a pyramidal horn antenna, given the lengths of the two sides as A and B, and aperture efficiency η_A?

Hint: $$G_D = \left(\frac{4\pi}{\lambda^2}\right) A_e = \left(\frac{4\pi}{\lambda^2}\right) (A\,B\,\eta_A)$$

15. Calculate the diameter of a dish antenna for resolving 1.5° at 150 MHz. Assume the aperture efficiency as 0.4.

Hint:

$$D' = 1.22\frac{\lambda}{\alpha_s} \qquad D = D'\sqrt{\eta_A}$$

Assignment Problems-V

16. In a single dish telescope design, the difference between on-source and off-source (cold sky) temperatures (incremental temperature) is found to be zero. What are the possible ways to make it positive?
Ans.: Reduce $T_{\rm sys}$, or increase D.

17. The incremental temperature is found negative in a single dish radio telescope. It is not possible to reduce $T_{\rm sys}$, or increase D. What will you do to make the telescope work?
Ans.: Use time integration.

18. In general terms, how is the far electric field related with the aperture electric field? Give an expression.
Hint:

$$E_{\rm Far}\,(\text{direction cosines}) = K_A\ \mathcal{F}\,[E_{\rm Near}(\text{rect. coords.})]$$

$$E_{\rm Far}(l,m) =_{-\infty}^{\infty} E_{\rm Near}\,(u,v)\ e^{-j2\pi lu} e^{-j2\pi m\,v} du\,dv$$

Assignment Problems-VI

19. An one dimensional antenna array uses 10 isotropic radiators as its elements. If the separation between the elements are d what is the array factor?

Hint: $A_n = A_1 e^{j(N-n)\,\beta d\cos\theta} = A_1 e^{j(N-n)\,\psi}$, where $\psi = \beta d \sin\theta$

$$K_{AF}(\theta) = A_1 \sum_{n=1}^{N} e^{j(N-n)\,\psi} = A_1 \frac{1 - e^{jN\,\psi}}{1 - e^{j\,\psi}} = A_1\, e^{j(N-1)\,\psi/2} \left[\frac{\sin(N\psi/2)}{\sin(\psi/2)}\right]$$

20. Describe the merits and demerits of a filled array.

21. Explain the grating array with a neat diagram.

22. How many cross and self products can be obtained from a grating array consisting of 30 elements?

Hint: For cross products:

$$n_c = {}^{n_a}C_2 = \frac{n_a\,(n_a - 1)}{2}$$

THANK YOU